Appendix B – Allocating Shares of Carrying Capacities (aSoCCs) under uncertainty

Erwan Ike de Bantel[a,*], Thibault Pirson[b], Gonzalo Puig-Samper[c], Jan Marcus Hartmann[d],
David Bol[b], Ghada Bouillass[a], Bernard Yannou[a], Marija Jankovic[a], Michael Hauschild[e,f]
[a] Université Paris-Saclay, CentraleSupélec, Laboratoire Genie Industriel, 91190 Gif-sur-Yvette, France [b] ICTEAM, Université catholique de Louvain, Louvain-la-Neuve, Belgium [c] Luxembourg Institute of Science and Technology, 5 Avenue des Hauts-Fourneaux, Esch-sur-Alzette, L-4362, Luxembourg [d] Institute of Technical Thermodynamics, RWTH Aachen University, Schinkelstraße 8, 52062 Aachen, Germany [e] Section for Quantitative Sustainability Assessment, Department of Environmental and Resource Engineering, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark
[f] Centre for Absolute Sustainability, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark
[*] Corresponding author: erwan.de-bantel@centralesupelec.fr.

Contents

uncasext_preprint_arXiv:2606.21465

Funding Information: The work of Erwan Ike de Bantel received organizational support of the "CircularIT Alliance" project. The work of Thibault Pirson is part of the SOIL project, which received funding from the European Union's Horizon Europe research and innovation program under the HORIZON-KDT-JU-2023-1-IA grant agreement N°101139785. Views and opinions expressed are however those of the authors only and do not necessarily reflect those of the European Union or CHIPS. Neither the European Union nor the granting authority can be held responsible for them.

This document details the allocation paths, accounting system boundaries, and functional units covered by the *UNCASExt* framework. It further specifies the underlying data sources, the allocation methods included, the methods used to project allocated shares, and the sources of uncertainty considered, including how they are propagated. Practical implementation is supported by the `pyaesa` Python package available at `https://github.com/AESAtoolkit/pyaesa`.

Notations

Symbol	**Definition**
MRIO matrices symbols follow pymrio terminology [Stadler, 2021].	
For all matrices, the first index denotes the row and the second denotes the column. This applies to notations of the form (row_a, row_b), (col_a, col_b)*; or* $row_a, (col_a, col_b)$*; or* row_a, col_a*; etc.*	
Reg	Set of all regions in the MRIO.
Sec	Set of all sectors in the MRIO.
FD	Set of all final demand categories.[a]
VA	Set of all value added categories in the factor inputs extension.[b]
E	Set of environmental impact categories in the environmental extensions (after applying LCIA method to obtain carrying capacities control variables units).
$r \in \mathrm{Reg}$	Generic region.
$s \in \mathrm{Sec}$	Generic sector.
r_c	Consuming region (direct consumption by final and intermediate demand).
r_f	Region in which final demand occurs.
(r_p, s_p)	Producing sector–region pair of interest.
(r_d, s_d)	Downstream buyer sector–region pair (a direct purchaser of the studied production outputs in the intermediate-transactions block).
(r_u, s_u)	Upstream supplier sector–region pair (a supplier whose outputs are mobilized to enable the studied production).
$y \in \mathrm{FD}$	Final demand category.
$v \in \mathrm{VA}$	Value added category.
$e \in \mathrm{E}$	Carrying capacity control variable category.
t	Studied year.
$rp_e(t)$	Responsibility period associated with the studied year t for impact category e, defined as the set of years over which cumulative environmental responsibility is evaluated.
$t^{\mathrm{rp}} \in rp_e(t)$	Generic year belonging to the responsibility period associated with the studied year t for impact category e.
$Z_{(r,s),(r',s')}(t)$	Intermediate flow from sector s in region r (row) to sector s' in region r' (column) at year t.
$\mathrm{Y}_{(r_p,s_p),(r_f,y)}(t)$	Final demand flow from sector s_p in region r_p (row) to final demand category y in region r_f (column) at year t.
$x_{(r,s)}(t)$	Total gross output of sector s in region r at year t.
$\hat{x}_{(r,s)}(t)^{-1}$	Inverse diagonal element of the gross output matrix for sector s in region r at year t.
$F_{v,(r,s)}(t)$	Factor production value added block representing the total amount of value added category v (row) generated by sector s in region r (column) to produce its total gross output $x_{(r,s)}(t)$ at year t.
$F_{e,(r,s)}(t)$	Factor production environmental extensions representing the total amount of environmental impact category e (row) generated by sector s in region r (column) to produce its total gross output $x_{(r,s)}(t)$ at year t.

Symbol	**Definition**
$S_{v,(r,s)}(t)$	Factor production value added block coefficients representing value added category v generated by sector s in region r per unit of its total gross output $x_{(r,s)}(t)$ at year t, with: $S_{v,(r,s)}(t) = F_{v,(r,s)}(t)\,\hat{x}_{(r,s)}(t)^{-1}.$
$S_{e,(r,s)}(t)$	Factor production environmental extensions coefficients representing environmental impact category e generated by sector s in region r per unit of its total gross output $x_{(r,s)}(t)$ at year t, with: $S_{e,(r,s)}(t) = F_{e,(r,s)}(t)\,\hat{x}_{(r,s)}(t)^{-1}.$
$F^{Y}_{v,(r_f,y)}(t)^{\mathrm{c}}$	Final demand factor production value added block associated directly with final demand category y in region r_f at year t, representing the total amount of value added category v (row) attributed to the final demand column (r_f, y).
$F^{Y}_{e,(r_f,y)}(t)^{\mathrm{c}}$	Final demand factor production environmental extensions associated directly with final demand category y in region r_f at year t, representing the total amount of environmental impact category e (row) attributed to the final demand column (r_f, y).
$S^{Y}_{v,(r_f,y)}(t)$	Final demand factor production value added block coefficients representing value added category v, associated directly with final demand category y in region r_f per unit of total final demand in column (r_f, y) at year t, with: $S^{Y}_{v,(r_f,y)}(t) = F^{Y}_{v,(r_f,y)}(t)\left(\sum_{r_p\in\mathrm{Reg}}\sum_{s_p\in\mathrm{Sec}} Y_{(r_p,s_p),(r_f,y)}(t)\right)^{-1}.$
$S^{Y}_{e,(r_f,y)}(t)$	Final demand factor production environmental extensions coefficients representing environmental impact category e, associated directly with final demand category y in region r_f per unit of total final demand in column (r_f, y) at year t, with: $S^{Y}_{e,(r_f,y)}(t) = F^{Y}_{e,(r_f,y)}(t)\left(\sum_{r_p\in\mathrm{Reg}}\sum_{s_p\in\mathrm{Sec}} Y_{(r_p,s_p),(r_f,y)}(t)\right)^{-1}.$
I	Identity matrix.
u	Column vector of ones with dimension equal to the number of region–sector pairs or final demand columns (used to sum elements).
$A_{(r_u,s_u),(r_p,s_p)}(t)$	Technical coefficient giving the intermediate input from sector s_u in region r_u (row) required per unit of output of sector s_p in region r_p (column) at year t, with: $A_{(r_u,s_u),(r_p,s_p)}(t) = Z_{(r_u,s_u),(r_p,s_p)}(t)\,\hat{x}_{(r_p,s_p)}(t)^{-1}.$
$L_{(r_u,s_u),(r_p,s_p)}(t)$	Leontief inverse coefficient giving the total (direct and indirect) production required from sector s_u in region r_u (row) per unit of output of sector s_p in region r_p (column) at year t, with: $L_{(r_u,s_u),(r_p,s_p)}(t) = [I - A(t)]^{-1}_{(r_u,s_u),(r_p,s_p)}.$
$M_{e,(r_p,s_p)}(t)$	Environmental impact multiplier for category e associated with output of (r_p, s_p) at year t (i.e., total, direct and indirect, requirement factors for one unit of output), with: $M_{e,(r_p,s_p)}(t) = \sum_{r_u\in\mathrm{Reg}}\sum_{s_u\in\mathrm{Sec}} S_{e,(r_u,s_u)}(t)\,L_{(r_u,s_u),(r_p,s_p)}(t),$

Symbol	Definition
$M_{v,(r_p,s_p)}(t)$	Value added multiplier for category v associated with output of (r_p, s_p) at year t (i.e., total, direct and indirect, requirement factors for one unit of output), with: $M_{v,(r_p,s_p)}(t) = \sum_{r_u \in \text{Reg}} \sum_{s_u \in \text{Sec}} S_{v,(r_u,s_u)}(t)\, L_{(r_u,s_u),(r_p,s_p)}(t)$
$B_{(r_p,s_p),(r_d,s_d)}(t)$	Input share coefficient, giving the share of the gross output of sector s_p in region r_p (row) delivered as intermediate input to sector s_d in region r_d (column) at year t, with: $B_{(r_p,s_p),(r_d,s_d)}(t) = \hat{x}_{(r_p,s_p)}(t)^{-1}\, Z_{(r_p,s_p),(r_d,s_d)}(t), \qquad \text{i.e.,} \qquad B(t) = \hat{x}(t)^{-1} Z(t).$
$G_{(r_p,s_p),(r_d,s_d)}(t)$	Ghosh inverse coefficient, giving the total (direct and indirect) downstream propagation weight from sector s_p in region r_p (row) to downstream sector s_d in region r_d (column) at year t, with: $G_{(r_p,s_p),(r_d,s_d)}(t) = [I - B(t)]^{-1}_{(r_p,s_p),(r_d,s_d)}\,,$ with G(t) also satisfying $G(t) = \hat{x}(t)\, L(t)\, \hat{x}(t)^{-1}.$
$P_r(t)$	Population of region r at year t.
$GDPcap_r(t)$	GDPppp per capita of region r at year t.[d]

---

[a] Final demand categories (FD) considered (all categories of Y): Final consumption expenditure by households; Final consumption expenditure by non-profit institutions serving households (NPISH); Final consumption expenditure by government; Gross fixed capital formation; Changes in inventories; Changes in valuables; Exports.
Final demand in MRIO tables extends beyond final consumption expenditure (FCE) because Y is defined to capture *all* non-intermediate uses of output. In addition to FCE, MRIOs include other final uses that absorb production but do not appear in intermediate demand Z and would therefore be omitted if Y were restricted to the FCE categories. Particularly, for AESA studies under CBA_{TD} system boundaries, excluding these additional components would imply that parts of sectors' outputs have no final absorber, even though total output is still accounted for in the environmental impact and value-added enacting metrics; the corresponding upstream requirements would then not be propagated to final users, biasing allocated shares.
Concretely, MRIO final demand includes, in addition to FCE: (i) investment demand (gross fixed capital formation), capturing purchases of capital goods that provide services over multiple years but are recorded as final demand in the accounting year; (ii) stock changes (changes in inventories and valuables), capturing production not immediately consumed within the period; and (iii) exports, capturing final demand by non-residents that is not assigned to a specific foreign region in the MRIO final-demand block.

[b] Value added categories considered (all categories of factor production value added block): Taxes less subsidies on products purchased; Other net taxes on production; Compensation of employees, wages, salaries, and employers social contributions: Low skilled, Medium skilled, and High skilled; Operating surplus: Consumption of fixed capital, Rents on land, Royalties on resources, and Remaining net operating surplus.

[c] MRIO tables provide a matrix F^Y of direct factor production (value added block and environmental extensions) recorded at final demand only. These entries capture flows not endogenised in the production system by sector category, for example, direct household emissions from heating and private transport, or land take for artificial surfaces [Stadler et al., 2018, Södersten et al., 2018].

[d] GDPppp = GDP adjusted for purchasing power parity, allowing GDPs to be compared across regions by accounting for differences in price levels. This form of GDP is selected as it is the only one available for future GDP projections provided by SSP scenarios [IIASA, 2024].

1 Allocation paths, system boundaries and functional units covered by UNCASExt

The following subsections present the allocation paths, accounting system boundaries, and functional units covered by the *UNCASExt* framework at each allocation level.

1.1 Allocation paths and accounting system boundaries

Figure 1 provides an overview of the allocation levels, allocation paths, and accounting system boundaries implemented in *UNCASExt*. All detailed definitions of each allocation method at each allocation level are provided in Section 3. Practical implementation is supported by the `pyaesa` Python package available at `https://github.com/AESAtoolkit/pyaesa`.

Reading guide and abbreviations. Figure 1 uses the terminology and abbreviations of [Puig-Samper et al., 2025] and [Bjørn et al., 2025]. Boxes indicate allocation levels L_1–L_5: L_1 (countries and groups of countries), L_2 (sectors), L_3 (companies), L_4 (goods and services), L_5 (persons). Solid boxes indicate levels that can be operationalized using shared open-access data sources, while dashed boxes indicate levels that require case study specific data.

Accounting system boundaries are defined as follows. CBA denotes consumption-based accounting (Scopes 1–2–3) and is specified for final demand (FD) or total demand (TD). PBA denotes production-based accounting (Scope 1) and is specified for total output (TO). The notation aSoCC denotes the allocated share of carrying capacity.

Allocation methods are obtained through the combination of a sharing principle (SP) (i.e, distributive justice principle) and an enacting metric (EM), which operationalizes this principle in quantitative terms [Bai et al., 2024, Puig-Samper et al., 2025]. The sharing principles are UT (utilitarian), AR (acquired rights), EG (egalitarian), PR (prioritarian), and PR-HR (historical responsibility variant of prioritarian). The enacting metrics are S (services delivered/produced, e.g., passenger-kilometers, kWh, etc.), E_e (environmental pressure for impact category e, corresponding to the carrying capacity control variable e), GVA (gross value added), and FD (final demand). The suffix "a" denotes an adjusted variant of the corresponding enacting metric (FDa, GVAa). Green labels indicate methods for which aSoCCs vary by carrying capacity control variable e, while black labels indicate methods providing the same aSoCC across control variables (independent).

1.1.1 Data requirements

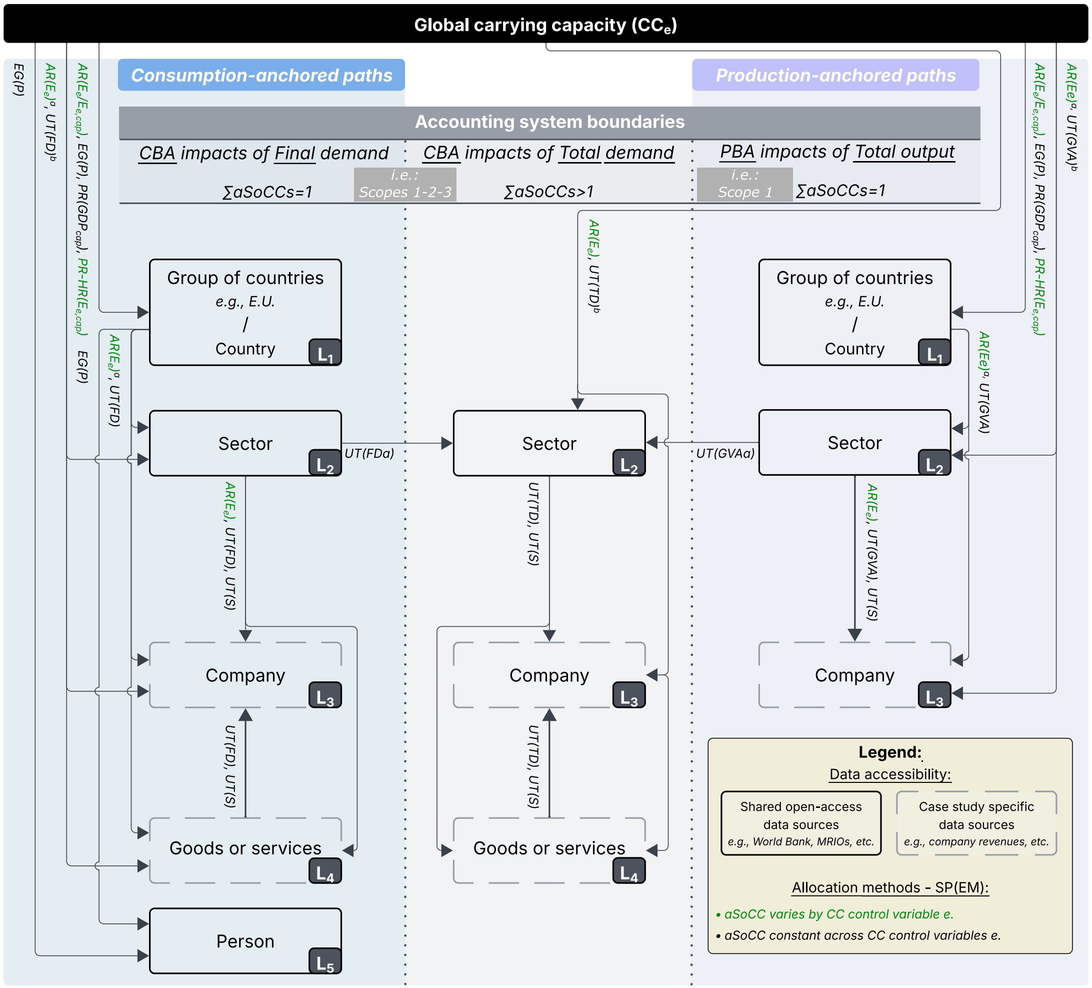
At L_1 and L_2, allocation can be implemented using shared open-access data sources (Section 2). The `pyaesa` Python package supports streamlined and reproducible data retrieval, harmonization, and processing across sources at L_1 and L_2.

At L_3 and L_4, case study specific data are required. Examples include company revenues to scale a sector-level allocation to a firm-level allocation by comparing company turnover to total sector output in the MRIO [Oosterhoff et al., 2023], or activity data (e.g., passenger-kilometers delivered) benchmarked against corresponding totals at the relevant MRIO level. L_3 and L_4 are not currently directly covered by `pyaesa`.

1.1.2 Definitions

Allocation levels and allocation paths. Allocation levels L_1–L_5 denote the available granularities at which an allocated share (aSoCC) can be estimated: L_1 (countries and groups of countries), L_2 (sectors), L_3 (companies), L_4 (goods and services), and L_5 (persons).

uncasext_preprint_arXiv:2606.21465



Abbreviations:

L.=allocation level; aSoCCs=allocated shares of carrying capacities; E.U.=European Union; PBA=production-based accounting; CBA=consumption-based accounting.

SP=sharing principle: UT=utilitarian; AR=acquired rights (grandfathering); EG=egalitarian; PR=prioritarian; HR=historical responsibility.
EM=enacting metric: GVA=gross value added; GVAa=adjusted gross value added (via the Leontief inverse); FD=final demand; FDa=adjusted final demand (via the Ghosh inverse); TD=total demand (final + intermediate); E_e=environmental pressure for impact category e; P=population; S=services delivered/produced.

Figure 1: Overview of the accounting system boundaries, allocation paths and methods covered by the *UNCASExt* framework across allocation levels. Unless stated otherwise (a), any allocation method at a given level can be combined with any allocation method at the next level along a chosen allocation path, indicated by arrows in the figure. All L_1 and L_2 allocation methods can be implemented in the Python package `pyaesa`. Adaptation of [Bjørn et al., 2025].

[a] *UT(FDa) and UT(GVAa) are adjusted variants of UT(FD) and UT(GVA) designed to reflect overlapping supply-chain propagations of utility in* CBA_{TD} *system boundaries. Therefore, they cannot be combined within* L_2 *with* $AR(E_e)$*. Such a combination is not aligned with the intent of adjusted metrics.*

An *allocation path* is the sequence of levels effectively traversed to allocate a share of carrying capacity from a higher-level reference, ultimately the global carrying capacity CC_e, to the assessed human activity. *UNCASExt* distinguishes *consumption-anchored paths* and *production-anchored paths* Figure 1). In both cases, an allocation path may traverse adjacent levels sequentially or may skip one or more intermediate levels, depending on the selection method.

Consistency between the ASR numerator and denominator. All AESA studies rely on the Absolute Sustainability Ratio (ASR), which compares the environmental burdens of a studied human activity with its allocated carrying capacity for each environmental category e:

$$ASR_e = \frac{IS_e}{\underbrace{aSoCC_e \times CC_e}_{aCC_e}}$$

where IS_e denotes the estimated environmental burden or impact score (result of Phase A) for environmental impact category e, $aSoCC_e$ the allocated share of carrying capacity assigned to the studied activity, CC_e the corresponding global carrying capacity, and aCC_e the allocated carrying capacity (result of Phase B). An $ASR_e \leq 1$ indicates that the activity can be considered absolute environmentally sustainable, as it operates within its allocated carrying capacity [Bjørn et al., 2020, Bjørn et al., 2025].

A fundamental methodological requirement is that the ASR numerator (Phase A) and denominator (Phase B) refer to the same scope [Bjørn et al., 2025]. As discussed in the main manuscript (*"Section 2.3 Scope inconsistencies between estimated environmental burdens and allocated carrying capacities"*), mismatches can arise from three main sources:

(i) *Impact pathway modeling mismatch* concerns the consistency between the methods used to characterize elementary flows in the ASR numerator and denominator. Environmental burdens quantified in Phase A and environmental enacting metrics used in Phase B (only for LCIA-based allocation methods) must rely on LCIA methods that are consistent with the selected carrying capacity control variables.

(ii) *Accounting system boundary mismatch* concerns whether both sides of the ASR are formulated under production-based accounting (PBA) or consumption-based accounting (CBA). In the ASR numerator (Phase A), PBA corresponds to direct environmental burdens of the studied activity (Scope 1), whereas CBA includes direct and indirect supply-chain burdens (Scopes 1, 2, and 3). In the ASR denominator (Phase B), the same distinction applies to economic and environmental enacting metrics: for instance, gross value added (GVA) is PBA because it represents value added directly generated by the producing activity, whereas final demand (FD) is CBA because it corresponds to the value delivered to final users, which encompasses all upstream value creation required to produce this value. A mismatch arises when the numerator and denominator do not use the same accounting system boundary, for example, when CBA burdens are compared with PBA enacting metrics, or conversely.

(iii) *Demand perimeter mismatch* concerns the distinction between final demand and total demand. Final demand only captures output delivered to final users, i.e., business-to-consumer (BtoC) activities, whereas total demand also includes intermediate demand between sectors, i.e., business-to-business (BtoB) activities. The ASR numerator and denominator should therefore represent the same demand perimeter. This issue is particularly relevant for activities that are partly or entirely BtoB, or mixed BtoC/BtoB, because using final demand only in the denominator excludes part of the activity represented in a total demand ASR numerator.

These mismatches can induce systematic bias because the ASR no longer compares estimated environmental burdens with allocated carrying capacities defined over the same scope. In the main manuscript, the accounting system boundary mismatch is quantified by comparing CBA total demand with PBA total output, yielding a median underallocation factor of 4.4× across all allocation methods implemented in `pyaesa` and all sector-region pairs available in the MRIO table EXIOBASE 3.10.2. The demand perimeter mismatch is quantified by comparing CBA total demand with CBA final demand, yielding a median underallocation factor of 4.7×.

UNCASExt and `pyaesa` address these risks by explicitly distinguishing accounting system boundaries and demand perimeters, as shown in Figure 1. The following subsection defines the corresponding functional units, while Section 3 provides the allocation equations used to compute the associated allocated shares.

1.2 Functional units definition by allocation level

A core contribution of *UNCASExt* is the explicit and harmonized definition of functional units covered across allocation levels. This clarification supports consistency between the ASR numerator (Phase A) and denominator (Phase B), thereby strengthening the interpretability of AESA results.

Table 2 provides a practical guide for AESA practitioners. The intended workflow is as follows. First, the practitioner identifies the functional unit in Table 2 that aligns with the study goal and scope, including the targeted accounting system boundary (PBA, CBA$*FD$, or CBA$*TD$). Second, the practitioner refers to Section 3, where allocation equations are provided for each functional unit and allocation level to compute the corresponding aSoCC. Third, the practitioner defines the set of allocation methods to be considered, either by using the full set applicable to the selected functional unit or by justifying a constrained subset based on the decision context and value choices. For the selected method set, the practitioner can either choose to propagate inter-method uncertainty within the Monte Carlo simulation or report independent results for each allocation method; other uncertainty sources, such as inter-MRIO, LCIA, or reference year uncertainty, can also be propagated.

To reduce implementation risks and foster harmonization, the `pyaesa` Python package provides standardized allocation processes for all functional units available at L_1 and L_2, including automated downloading and processing of the open-access data sources required to compute the allocation equations.

Table 2: Functional units covered by UNCASExt across allocation levels and accounting system boundaries.

Allocation level	**Functional unit**	**Accounting system boundaries**	**Code**
L_1	Final demand of goods and services in region(s) r_f in year t	$\mathrm{CBA_{FD}}$	$L_{1.a}$
L_1	Total production of goods and services by producing region(s) r_p in year t	PBA	$L_{1.b}$
L_2	Total production of goods and services by sector s_p in producing region(s) r_p directly supplied to final demand worldwide in year t	$\mathrm{CBA_{FD}}$	$L_{2.a.a}$
L_2	Total production of goods and services by sector s_p in producing region(s) r_p in year t	$\mathrm{CBA_{TD}}$	$L_{2.a.b}$
L_2	Total production of goods and services by sector s_p in producing region(s) r_p in year t	PBA	$L_{2.a.c}$
L_2	Total production of goods and services by sector s_p in producing region(s) r_p directly supplied to final demand in region(s) r_f in year t	$\mathrm{CBA_{FD}}$	$L_{2.b.a}$
L_2	Total production of goods and services by sector s_p in producing region(s) r_p directly supplied to total demand in region(s) r_c in year t	$\mathrm{CBA_{TD}}$	$L_{2.b.b}$
L_2	Final demand in region(s) r_f in year t of goods and services produced by sector s_p	$\mathrm{CBA_{FD}}$	$L_{2.c.a}$

Allocation level	Functional unit	Accounting system boundaries	Code
L_2	Total demand in region(s) r_c in year t of goods and services produced by sector s_p	$\mathrm{CBA}_{\mathrm{TD}}$	$L_{2.c.b}$
L_3	Total production of goods and services by company b_p in producing region(s) r_p directly supplied to final demand worldwide in year t	$\mathrm{CBA}_{\mathrm{FD}}$	$L_{3.a.a}$
L_3	Total production of goods and services by company b_p in producing region(s) r_p in year t	$\mathrm{CBA}_{\mathrm{TD}}$	$L_{3.a.b}$
L_3	Total production of goods and services by company b_p in producing region(s) r_p in year t	PBA	$L_{3.a.c}$
L_3	Total production of goods and services by company b_p in producing region(s) r_p directly supplied to final demand in region(s) r_f in year t	$\mathrm{CBA}_{\mathrm{FD}}$	$L_{3.b.a}$[a]
L_3	Total production of goods and services by company b_p in producing region(s) r_p[a] directly supplied to total demand in region(s) r_c in year t	$\mathrm{CBA}_{\mathrm{TD}}$	$L_{3.b.b}$
L_4	Total production of a specific good or service g in producing region(s) r_p directly supplied to final demand worldwide in year t (*incl. per unit intensity thresholds*)	$\mathrm{CBA}_{\mathrm{FD}}$	$L_{4.a.a}$
L_4	Total production of a specific good or service g in producing region(s) r_p in year t (*incl. per unit intensity thresholds*)	$\mathrm{CBA}_{\mathrm{TD}}$	$L_{4.a.b}$
L_4	Total production of a specific good or service g in producing region(s) r_p directly supplied to final demand in region(s) r_f in year t (*incl. per unit intensity thresholds*)	$\mathrm{CBA}_{\mathrm{FD}}$	$L_{4.b.a}$[a]
L_4	Total production of a specific good or service g in producing region(s) r_p[a] directly supplied to total demand in region(s) r_c in year t (*incl. per unit intensity thresholds*)	$\mathrm{CBA}_{\mathrm{TD}}$	$L_{4.b.b}$
L_4	Final demand in region(s) r_f in year t of a specific good or service g (*incl. per unit intensity thresholds*)	$\mathrm{CBA}_{\mathrm{FD}}$	$L_{4.c.a}$
L_4	Total demand in region(s) r_c in year t of a specific good or service g (*incl. per unit intensity thresholds*)	$\mathrm{CBA}_{\mathrm{TD}}$	$L_{4.c.b}$
L_5	Final demand of goods and services by a single person in region(s) r_f in year t	$\mathrm{CBA}_{\mathrm{FD}}$	$L_{5.a}$

FD = final demand; TD = total demand (final + intermediate); PBA = production-based accounting; $\mathrm{CBA}_{\mathrm{FD}}$ = consumption-based accounting of final demand; $\mathrm{CBA}_{\mathrm{TD}}$ = consumption-based accounting of total demand.

[a] $L_{3/4.a.a}$ and $L_{3/4.a.b}$ do not specify the region(s) of final/total demand r_f/r_c, i.e., the destination markets of the studied company or studied good or service are unknown. Therefore, under allocation paths that first pass through L_2, the underlying assumption, following [Oosterhoff et al., 2023], is that the regional sales distribution of the studied company, good, or service is identical to that of the corresponding producing sector-region pair to which it is linked in the MRIO.

2 Data sources

This section summarizes the data sources used to compute the enacting metrics implemented in *UNCASExt* and `pyaesa`. For external data, population and GDP rely on World Bank World Development Indicators (WDI) data through 2024 [WorldBank, 2025], complemented by prospective SSP trajectories from the IIASA SSP Scenario Explorer from 2025 onward [International Institute for Applied Systems Analysis, nd]. In the World Bank database, Taiwan is listed as part of China, whereas it is reported as a standalone entity in MRIO tables. To ensure consistency across datasets, Taiwan data are obtained from the International Monetary Fund World Economic Outlook (WEO) database [InternationalMonetaryFund, 2025] and subtracted from China's World Bank time series.

For MRIO data, the data sources used are EXIOBASE 3.10.2 [Stadler et al., 2018, Stadler et al., 2026] and OECD-ICIO v2025 [OECD, 2023]. EXIOBASE 3.10.2 covers 1995–2024 (2023 and 2024 are nowcasted), while OECD-ICIO v2025 covers 1995–2022. MRIO-based enacting metrics include final demand (FD), total demand (TD), gross value added (GVA), adjusted final demand (FDa), adjusted gross value added (GVAa), and environmental pressures E_e (the latter being available only in EXIOBASE).

The following table summarizes the data sources and their temporal coverage:

Enacting metric (EM)	Retrospective years	Prospective years
FD, GVA, E *MRIO*	**Sources:** EXIOBASE v3.10.2[a][Stadler et al., 2018, Stadler et al., 2026], OECD ICIO v2025 [OECD, 2025]. **Years:** Economic metrics: 1995–2024.[a] Environmental pressure metrics: 1995–2024.[a]	Not available for prospective years. Prospective years included in `pyaesa` for economic enacting metrics via regression or historical reuse (see Section. 5)
Pop, GDP *External data, no MRIO*	**Source:** World Bank World Development Indicators (WDI) [WorldBank, 2025].[c] **Years:** 1995–2024.	**Source:** IIASA SSP Scenario Explorer [International Institute for Applied Systems Analysis, nd].[b] **Years:** 2025–2100. **Scenarios:** SSP1, SSP2, SSP3, SSP4 and SSP5.

FD = final demand; GVA = gross value added; Pop = population; GDP = gross domestic product; SSP = Shared Socioeconomic Pathway; IIASA = International Institute for Applied Systems Analysis; WDI = World Development Indicators; IMF = International Monetary Fund; WEO = World Economic Outlook; LCA = life-cycle assessment.

[a] 2023 and 2024 are only available in EXIOBASE v3.10.2 as nowcasted years.

[b] SSP data are provided at five-year intervals. Intermediate years are obtained by linear interpolation.

[c] Missing data for a few years and countries are reconstructed using log-linear regression. In the World Bank database, Taiwan is listed as part of China, whereas it is reported as a standalone entity in MRIO tables and SSP scenarios. To ensure consistency across datasets, Taiwan data are obtained from the International Monetary Fund World Economic Outlook (WEO) database [InternationalMonetaryFund, 2025] and subtracted from China's World Bank data. It is not possible to rely exclusively on International Monetary Fund data for all countries, as too many data points are missing across the period 1995–2024 compared to World Bank data.

3 Allocation methods covered by *UNCASExt*: mathematical expressions by allocation level and functional unit

This section provides the mathematical expressions of the allocation methods operationalized in *UNCASExt*, classified by allocation level and, within each level, by the functional units (FUs) defined in Table 2. For each allocation level, we first provide the equations required to compute MRIO-based enacting metrics within the relevant accounting system boundaries, based on `pymrio` notations [Stadler, 2021], and then report the corresponding allocation equations for each functional unit. The `pyaesa` Python package supports end-to-end reproducibility, from data downloading and processing to the computation of enacting metrics and allocated shares for all L_1 and L_2 methods reported in this section.

3.1 Allocation level L_1: countries and groups of countries

At L_1, allocation equations are computed for countries (or groups of countries) under two accounting system boundaries depending on the functional unit: FU $L_{1.b}$ targets total production by producing region(s) r_p (PBA; Scope 1), whereas FU $L_{1.a}$ targets final demand by consuming region(s) r_f (CBA$_{\text{FD}}$; Scopes 1–2–3 restricted to final demand).

3.1.1 L_1 MRIO-based enacting metrics

This subsection specifies the computation of the MRIO-based enacting metrics used at L_1 under the two relevant accounting system boundaries. The environmental enacting metric is denoted $E^{\cdot}$, where the superscript "$\cdot$" is a placeholder for the accounting system boundaries. At L_1, $\cdot \in \{\text{PBA}, \text{CBA}_{FD}\}$: E^{PBA} represents production-based (Scope 1) impacts occurring in producing regions r_p, whereas $E^{\text{CBA}_{FD}}$ represents consumption-based (Scopes 1–2–3) impacts generated worldwide to satisfy final demand in regions

r_f. The corresponding MRIO expressions (including per-capita and cumulative-per-capita variants used by prioritarian and historical responsibility methods) are reported in Table 4.

Table 4: L_1 MRIO-based environmental enacting metrics

Enacting metric	**Definition**	**Equation**
$E^{\mathrm{CBA}_{FD}}_{e,r_f}(t)$	$\mathrm{CBA_{FD}}$ environmental impact for category e generated worldwide to satisfy *final demand* in region r_f in year t.	$E^{\mathrm{CBA}_{FD}}_{e,r_f}(t) = \sum_{r_u \in \mathrm{Reg}} \sum_{s_u \in \mathrm{Sec}} S_{e,(r_u,s_u)}(t) \sum_{r_p \in \mathrm{Reg}} \sum_{s_p \in \mathrm{Sec}} L_{(r_u,s_u),(r_p,s_p)}(t) \cdot \sum_{y \in FD} Y_{(r_p,s_p),(r_f,y)}(t) \; + \; F^{Y}_{e,r_f}(t) = D^{cba,reg}_{e,r_f}(t).$
$E^{\mathrm{PBA}}_{e,r_p}(t)$	PBA (Scope 1) environmental impact for category e occurring in producing region r_p in year t.	$E^{\mathrm{PBA}}_{e,r_f}(t) = \sum_{s_p \in Sec} S_{e,(r_p,s_p)}(t)\, x_{(r_p,s_p)}(t) \; + \; F^{Y}_{e,r_p}(t) = D^{pba,reg}_{e,r_p}(t)$
$E\mathrm{cap}^{\cdot}_{e,r}(t)$	Per-capita environmental impact.	$E\mathrm{cap}^{\cdot}_{e,r}(t) = \frac{E^{\cdot}_{e,r}(t)}{P_r(t)}$
$E\mathrm{cap,cum}^{\cdot}_{e,r}(t)$	Cumulative per-capita impact over the responsibility period of category e (Appendix B), where $\cdot \in \{\mathrm{PBA}, \mathrm{CBA}_{FD}\}$.	$E\mathrm{cap,cum}^{\cdot}_{e,r}(t) = \sum_{t^{\mathrm{rp}} \in rp_e(t)} \frac{E^{\cdot}_{e,r}(t^{\mathrm{rp}})}{P_r(t^{\mathrm{rp}})}.$
$FD_{r_f}(t)^{\mathrm{a}}$	Final demand of region r_f in year t.	$FD_{r_f}(t) = \sum_{s_p \in \mathrm{Sec}} \sum_{r_p \in \mathrm{Reg}} \sum_{y \in FD} Y_{(r_p,s_p),(r_f,y)}(t) = \sum_{v \in VA} D^{cba,reg}_{v,r_f}(t)$
$GVA_{r_p}(t)^{\mathrm{a}}$	Direct GVA generated in region r_p in year t.	$GVA_{r_p}(t) = \sum_{s_p \in \mathrm{Sec}} \sum_{v \in VA} F_{v,(r_p,s_p)}(t) = \sum_{v \in VA} D^{pba,reg}_{v,r_p}(t)$

PBA = production-based accounting; $\mathrm{CBA_{FD}}$ = consumption-based accounting of final demand; cap = per-capita; cap,cum = cumulative per-capita; FD = final demand; GVA = gross value added.

[a] $FD_{r_f}(t)$ and $GVA_{r_p}(t)$ are not used by any L_1 allocation method but are used at L_2 for one-step allocation methods.

3.1.2 L_1 allocation equations by functional unit

At L_1, *UNCASExt* includes the country-level allocation methods of the original UNCASE framework [Puig-Samper et al., 2025], with two updates that align with recent method inventories [Yang and Paulillo, 2025, Paulillo et al., 2026].

First, *UNCASExt* replaces PR(E_{cap}) as implemented in UNCASE with the responsibility-adjusted prioritarian method PR–HR($\mathrm{E}_{\mathrm{cap},r}$), i.e., a prioritarian scaling factor based on historical responsibility rather than on the studied-year impact level. This modification reflects that PR–HR requires a cumulative responsibility term over a defined responsibility period, which is not captured when metrics are measured for only a single year, as in the original UNCASE [Puig-Samper et al., 2025].

Second, *UNCASExt* includes AR($\mathrm{E}_{\mathrm{cap}}$) [Yang and Paulillo, 2025, Paulillo et al., 2026] as an additional option for computing acquired rights at L_1, complementing the standard acquired-rights formulation in the UNCASE setting. This has implications for prospective studies when future populations are projected.

Table 5 reports the SP(EM) allocation equations covered by *UNCASExt* at L_1 for both functional units $L_{1.b}$ (PBA) and $L_{1.a}$ (CBA_{FD}). The SP(EM) structures are identical across the two functional units; only the indexing and the accounting scope differ, i.e., (r_p, E^{PBA}) for FU $L_{1.b}$ versus $(r_f, E^{\mathrm{CBA}_{FD}})$ for FU $L_{1.a}$.

Table 5: L_1 allocation methods covered by UNCASExt

SP(EM)	**Definition**	**Equation**	**Original UNCASE**	**References**
$\boldsymbol{EG}\textbf{(Pop)}_r(t)$	Equal per capita: all individuals worldwide receive the same share in year t.	$\dfrac{P_r(t)}{\sum_{r' \in Reg} P_{r'}(t)}$	✓	[Lucas et al., 2020, Paulillo et al., 2026, Puig-Samper et al., 2025, Yang and Paulillo, 2025]
$\boldsymbol{PR}\textbf{(GDPcap)}_r(t)$	Ability to pay: inversely proportional to GDP per capita in year t.	$\dfrac{1}{GDPcap_r(t)} \times \dfrac{P_r(t)}{\sum_{r' \in Reg} \dfrac{P_{r'}(t)}{GDPcap_{r'}(t)}}$	✓	[Gebara and Laurent, 2023, Hjalsted et al., 2021, Paulillo et al., 2026, Puig-Samper et al., 2025, Verhaeghe et al., 2024, Yang and Paulillo, 2025]
$\boldsymbol{PR\text{-}HR}(\boldsymbol{E}\textbf{cap}, \textbf{cum}^{\cdot})_{e,r}(t)$	Historical responsibility: inversely proportional to cumulative per-capita impacts over the responsibility period $rp_e(t)$.	$\dfrac{1}{E\text{cap}, \text{cum}^{\cdot}_{e,r}(t)} \times \dfrac{P_r(t)}{\sum_{r' \in \text{Reg}} \dfrac{P_{r'}(t)}{E\text{cap}, \text{cum}^{\cdot}_{e,r'}(t)}}$	✗	[Lucas et al., 2020, Paulillo et al., 2026, Yang and Paulillo, 2025]
$\boldsymbol{AR}(\boldsymbol{E}^{\cdot})_{e,r}(t^{ar})$	Grandfathering: proportional to impacts in reference year t^{ar}.	$\dfrac{E^{\cdot}_{e,r}(t^{ar})}{\sum_{r' \in Reg} E^{\cdot}_{e,r'}(t^{ar})}$	✓	[Hjalsted et al., 2021, Lucas et al., 2020, Puig-Samper et al., 2025, van den Berg et al., 2020, Verhaeghe et al., 2024]
$\boldsymbol{AR}(\boldsymbol{E}\textbf{cap}^{\cdot})_{e,r}(t^{ar})$	Grandfathering (per-capita) in reference year t^{ar}, weighted by population at year t.	$\dfrac{E\text{cap}^{\cdot}_{e,r}(t^{ar})\, P_r(t)}{\sum_{r' \in \text{Reg}} E\text{cap}^{\cdot}_{e,r'}(t^{ar})\, P_{r'}(t)}$	✗	[Paulillo et al., 2026, Yang and Paulillo, 2025]

SP = sharing principle; EM = enacting metric; EG = egalitarian; PR = prioritarian; AR = acquired rights (grandfathering); HR = historical responsibility; Pop = population; cap = per-capita; cap,cum = cumulative per-capita; PBA = production-based accounting; CBA_{FD} = consumption-based accounting of final demand.

The superscript "·" is a placeholder for the accounting system boundaries of the environmental burdens E, impact category e.
$E^{\cdot} \in \{\text{PBA}, \text{CBA}_{FD}\}$.
For functional unit $L_{1.a}$ (CBA_{FD}), $r = r_f$ (region where final demand occurs).
For functional unit $L_{1.b}$ (PBA), $r = r_p$ (region of production).

3.1.3 Prioritarian sharing principles: equivalence between [Hjalsted et al., 2021], [Paulillo et al., 2026] and the factorized forms of [Yang and Paulillo, 2025]

The core idea of prioritarian approaches is to introduce a distortion in the egalitarian sharing principle by giving higher weight to a subgroup based on a specific criterion [Gebara and Laurent, 2023]. It can be, for example, to favor low-impacting countries based on their current environmental impacts per-capita levels (e.g., higher shares given to low-impacting countries relative to the global mean), or their ability to pay (based on GDP per-capita) [Gebara and Laurent, 2023, Hjalsted et al., 2021, Paulillo et al., 2026, Yang and Paulillo, 2025].

[Hjalsted et al., 2021] γ and ϵ based allocation factors vs. [Yang and Paulillo, 2025] factorized forms

Prioritarian equations in [Hjalsted et al., 2021] introduce a *priority factor* $\gamma(t)$ applied to a given enacting metric (e.g., environmental impacts or GDP) to favor countries with the lowest per capita value of the metric under consideration. However, the use of a priority factor necessitates the addition of a scaling factor $\epsilon(t)$ to ensure that the sum of all region shares sums to one [Hjalsted et al., 2021]. [Hjalsted et al., 2021] uses a priority factor based on the world average value of a given enacting metric *EM*, to which the value of the country r is compared. If the country's metric is higher (resp. lower) than the world average, its share is decreased (resp. increased). The general equation is thus as follows:

$$aSoCC^{PR}_{EM,r}(t) = \frac{P_r(t)}{P_W(t)} \times \gamma_{EM,r}(t) \times \varepsilon_{EM}(t) \tag{1}$$

with,

$$\begin{cases} \gamma_{EM,r}(t) = \left(\dfrac{EMcap_r(t)}{EMcap_W(t)}\right)^{-1} = \dfrac{EM_W(t)}{P_W(t)} \times \dfrac{P_r(t)}{EM_r(t)} \\ \varepsilon_{EM}(t) = \dfrac{(P_W(t))^2}{EM_W(t)} \times \left(\displaystyle\sum_{r'\in Reg} \dfrac{(P'_r(t))^2}{EM'_r(t)}\right)^{-1} \end{cases} \tag{2}$$

where $EMcap_W(t)$ denotes the world average per capita value of the enacting metric EM at time (t), and $P_W(t) = \sum_{r'\in Reg} P'_r(t)$ denotes total world population.

Equation (1) clearly shows that the prioritarian dimension is added on top of the egalitarian equal per-capita allocation, which justifies the squared population term appearing in the scaling factor $\varepsilon_{EM}(t)$ in Equation (2).

[Hjalsted et al., 2021] prioritarian approach is numerically identical the the factorized form introduced by [Yang and Paulillo, 2025]:

$$\text{[Hjalsted et al., 2021]: } aSoCC^{PR}_{EM,r}(t) = \frac{P_r(t)}{P_W(t)} \times \gamma_{EM,r}(t) \times \varepsilon_{EM}(t),$$

Substituting $\gamma_{EM,r}(t)$ and $\varepsilon_{EM}(t)$ gives:

$$\begin{aligned} aSoCC^{PR}_{EM,r}(t) &= \frac{P_r(t)}{P_W(t)} \left(\frac{EM_W(t)}{P_W(t)} \times \frac{P_r(t)}{EM_r(t)}\right) \left(\frac{(P_W(t))^2}{EM_W(t)} \times \left(\sum_{r'\in Reg} \frac{(P_{r'}(t))^2}{EM_{r'}(t)}\right)^{-1}\right) \\ &= \frac{(P_r(t))^2}{EM_r(t)} \times \left(\sum_{r'\in Reg} \frac{(P_{r'}(t))^2}{EM_{r'}(t)}\right)^{-1} \\ &= \frac{\dfrac{(P_r(t))^2}{EM_r(t)}}{\displaystyle\sum_{r'\in Reg} \dfrac{(P_{r'}(t))^2}{EM_{r'}(t)}}. \end{aligned} \tag{3}$$

Finally, using $EM_r(t) = EMcap_r(t)\, P_r(t)$:

$$\begin{aligned}
aSoCC^{PR}_{EM,r}(t) &= \frac{\dfrac{(P_r(t))^2}{EMcap_r(t)\, P_r(t)}}{\displaystyle\sum_{r' \in Reg} \dfrac{(P_{r'}(t))^2}{EMcap_{r'}(t)\, P_{r'}(t)}} \\
&= \frac{\dfrac{P_r(t)}{EMcap_r(t)}}{\displaystyle\sum_{r' \in Reg} \dfrac{P_{r'}(t)}{EMcap_{r'}(t)}} \\
&= \frac{1}{EMcap_r(t)} \times \frac{P_r(t)}{\displaystyle\sum_{r' \in Reg} \dfrac{P_{r'}(t)}{EMcap_{r'}(t)}}.
\end{aligned} \tag{4}$$

This corresponds to the factorized form of [Yang and Paulillo, 2025] adopted in Table 5.

[Paulillo et al., 2026] α based allocation factors vs. [Yang and Paulillo, 2025] factorized forms

The α based allocation factors proposed by [Paulillo et al., 2026] for the ability to pay principle $PR(GDPcap)$, the responsibility based principle $PR\text{-}HR(Ecap, cum)$ and the acquired rights principle $AR(Ecap)$ are algebraically equivalent to the factorized forms of [Yang and Paulillo, 2025] adopted in this appendix.

In the following equations, the subscript W denotes the world aggregate, for example $P_W(t) = \sum_{r' \in Reg} P_{r'}(t)$ and $GDPcap_W(t)$ the corresponding world average GDP per capita.

Ability to pay $PR(GDPcap)$

In [Paulillo et al., 2026], the ability to pay allocation coefficient for country r is written as:

$$PR(GDPcap)_r(t) = \frac{P_r(t)}{P_W(t)} \times \left(\frac{GDPcap_r(t)}{GDPcap_W(t)}\right)^{-1} \times \alpha_{\text{AtP}}(t), \tag{5}$$

with

$$\alpha_{\text{AtP}}(t) = \frac{P_W(t)}{GDPcap_W(t) \displaystyle\sum_{r' \in Reg} \dfrac{P_{r'}(t)}{GDPcap_{r'}(t)}}. \tag{6}$$

Substituting $\alpha_{\text{AtP}}(t)$ into the first expression gives:

$$\begin{aligned}
PR(GDPcap)_r(t) &= \frac{GDPcap_W(t)}{GDPcap_r(t)}\, \frac{P_r(t)}{P_W(t)}\, \frac{P_W(t)}{GDPcap_W(t) \displaystyle\sum_{r' \in Reg} \dfrac{P_{r'}(t)}{GDPcap_{r'}(t)}} \\
&= \frac{\dfrac{P_r(t)}{GDPcap_r(t)}}{\displaystyle\sum_{r' \in Reg} \dfrac{P_{r'}(t)}{GDPcap_{r'}(t)}} \\
&= \frac{1}{GDPcap_r(t)} \times \frac{P_r(t)}{\displaystyle\sum_{r' \in Reg} \dfrac{P_{r'}(t)}{GDPcap_{r'}(t)}}.
\end{aligned} \tag{7}$$

This corresponds to the factorized form of [Yang and Paulillo, 2025] adopted in Table 5.

Historical responsibility $PR\text{-}HR(Ecap, cum)$

In [Paulillo et al., 2026], the historical responsibility allocation coefficient for country r and environmental impact category e is written as:

$$PR\text{-}HR(Ecap, cum)_{e,r}(t) = \frac{Ecap, cum_{e,W}(t)}{Ecap, cum_{e,r}(t)} \times \frac{P_r(t)}{P_W(t)} \times \alpha_{\mathrm{HR},e}(t), \tag{8}$$

with

$$\alpha_{\mathrm{HR},e}(t) = \frac{P_W(t)}{Ecap, cum_{e,W}(t) \sum_{r' \in Reg} \frac{P_{r'}(t)}{Ecap, cum_{e,r'}(t)}}. \tag{9}$$

Substituting $\alpha_{\mathrm{HR},e}(t)$ into the first expression gives:

$$\begin{aligned} PR\text{-}HR(Ecap, cum)_{e,r}(t) &= \frac{Ecap, cum_{e,W}(t)}{Ecap, cum_{e,r}(t)} \frac{P_r(t)}{P_W(t)} \frac{P_W(t)}{Ecap, cum_{e,W}(t) \sum_{r' \in Reg} \frac{P_{r'}(t)}{Ecap, cum_{e,r'}(t)}} \\ &= \frac{\frac{P_r(t)}{Ecap, cum_{e,r}(t)}}{\sum_{r' \in Reg} \frac{P_{r'}(t)}{Ecap, cum_{e,r'}(t)}} \\ &= \frac{1}{Ecap, cum_{e,r}(t)} \times \frac{P_r(t)}{\sum_{r' \in Reg} \frac{P_{r'}(t)}{Ecap, cum_{e,r'}(t)}}. \end{aligned} \tag{10}$$

This corresponds to the factorized form of [Yang and Paulillo, 2025] adopted in Table 5.

Acquired rights $AR(Ecap)$

In [Paulillo et al., 2026], the acquired rights (grandfathering) allocation coefficient for country r and environmental impact category e is written as:

$$AR(Ecap)_{e,r}(t) = \frac{Ecap_{e,r}(t^{ar})}{Ecap_{e,W}(t^{ar})} \times \frac{P_r(t)}{P_W(t)} \times \alpha_{\mathrm{AR},e}(t), \tag{11}$$

with

$$\alpha_{\mathrm{AR},e}(t) = \frac{Ecap_{e,W}(t^{ar})\, P_W(t)}{\sum_{r' \in Reg} Ecap_{e,r'}(t^{ar})\, P_{r'}(t)}. \tag{12}$$

Substituting $\alpha_{\mathrm{AR},e}(t)$ into the first expression gives:

$$\begin{aligned} AR(Ecap)_{e,r}(t) &= \frac{Ecap_{e,r}(t^{ar})}{Ecap_{e,W}(t^{ar})} \frac{P_r(t)}{P_W(t)} \frac{Ecap_{e,W}(t^{ar})\, P_W(t)}{\sum_{r' \in Reg} Ecap_{e,r'}(t^{ar})\, P_{r'}(t)} \\ &= \frac{Ecap_{e,r}(t^{ar})\, P_r(t)}{\sum_{r' \in Reg} Ecap_{e,r'}(t^{ar})\, P_{r'}(t)}. \end{aligned} \tag{13}$$

This corresponds to the factorized form of [Yang and Paulillo, 2025] adopted in Table 5.

3.2 Allocation level L_2: sectors

At L_2, aSoCCs are computed for sectors, either as producing sector–region pairs (r_p, s_p) (production-anchored functional units) or as sectors s_p serving directly final demand in a region r_f or serving directly total demand in a region r_c (consumption-anchored functional units), depending on the FU in Table 2. As at L_1, the convention is that the superscript "$\cdot$" is a placeholder for the accounting system boundaries. At L_2, $\cdot \in \{\mathrm{PBA}, \mathrm{CBA}_{FD}, \mathrm{CBA}_{TD}\}$ depending on the FU.

3.2.1 L_2 MRIO-based enacting metrics

This subsection specifies the MRIO-based enacting metrics used at L_2 under the accounting system boundaries relevant to the considered FU. Environmental enacting metrics are denoted $E^{\cdot}$, with E^{PBA} for production-based accounting, $E^{\mathrm{CBA}_{FD}}$ for consumption-based accounting restricted to final demand, and $E^{\mathrm{CBA}_{TD}}$ for consumption-based accounting targeting total demand (i.e., total output at L_2). In addition, utilitarian enacting metrics are denoted by (i) FD (final demand) and (ii) GVA (gross value added). Table 6 summarizes these L_2 enacting metrics.

Table 6: L_2 MRIO-based enacting metrics.

Enacting metric	**Definition**	**Equation**
$FD_{(r_p,s_p),r_f}(t)$	Final demand supplied by producing sector-region pair (r_p, s_p) to r_f in year t.	$FD_{(r_p,s_p),r_f}(t) = \sum_{y\in FD} Y_{(r_p,s_p),(r_f,y)}(t)$
$FD_{(r_p,s_p)}(t)$	Worldwide final demand supplied by producing sector-region pair (r_p, s_p) in year t.	$FD_{(r_p,s_p)}(t) = \sum_{r_f\in Reg} FD_{(r_p,s_p),r_f}(t)$
$FD_{(r_f,s_p)}(t)$	Final demand supplied by sector s_p to r_f in year t.	$FD_{(r_f,s_p)}(t) = \sum_{r_p\in\mathrm{Reg}}\sum_{y\in FD} Y_{(r_p,s_p),(r_f,y)}(t) = \sum_{v\in VA} D^{cba}_{v,(r_f,s_p)}(t)$
$E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p),r_f}(t)$	Quantity of environmental impact category e embodied in the outputs of producing sector-region pair (r_p, s_p) *directly supplied to final demand* in r_f in year t.	$E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p),r_f}(t) = \sum_{r_u\in\mathrm{Reg}}\sum_{s_u\in\mathrm{Sec}} S_{e,(r_u,s_u)}(t)\, L_{(r_u,s_u),(r_p,s_p)}(t) \cdot \sum_{y\in FD} Y_{(r_p,s_p),(r_f,y)}(t)$ $= M_{e,(r_p,s_p)}(t) \sum_{y\in FD} Y_{(r_p,s_p),(r_f,y)}(t)$
$E^{\mathrm{CBA}_{TD}}_{(e,r_p,s_p),r_c}(t)$	Quantity of environmental impact category e embodied in the outputs of producing sector-region pair (r_p, s_p) *directly supplied to total demand* (final + intermediate) in r_c in year t.	$E^{\mathrm{CBA}_{TD}}_{(e,r_p,s_p),r_c}(t) = \sum_{r_u\in\mathrm{Reg}}\sum_{s_u\in\mathrm{Sec}} S_{e,(r_u,s_u)}(t)\, L_{(r_u,s_u),(r_p,s_p)}(t) \cdot \left(\sum_{y\in FD} Y_{(r_p,s_p),(r_c,y)}(t) + \sum_{s_c\in\mathrm{Sec}} Z_{(r_p,s_p),(r_c,s_c)}(t)\right)$ $= E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p),r_c}(t) + M_{e,(r_p,s_p)}(t) \sum_{s_c\in\mathrm{Sec}} Z_{(r_p,s_p),(r_c,s_c)}(t)$
$E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p)}(t)$	Quantity of environmental impact category e embodied in the outputs of producing sector-region pair (r_p, s_p) *directly supplied to final demand worldwide* in year t.	$E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p)}(t) = \sum_{r_f\in\mathrm{Reg}} E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p),r_f}(t)$
$E^{\mathrm{CBA}_{TD}}_{(e,r_p,s_p)}(t)$	Quantity of environmental impact category e embodied in the outputs of producing sector-region pair (r_p, s_p) *directly supplied to total demand* (final + intermediate) *worldwide* in year t.	$E^{\mathrm{CBA}_{TD}}_{(e,r_p,s_p)}(t) = \sum_{r_c\in\mathrm{Reg}} E^{\mathrm{CBA}_{TD}}_{(e,r_p,s_p),r_c}(t)$
$E^{\mathrm{CBA}_{FD}}_{e,(r_f,s_p)}(t)$	Quantity of environmental impact category e embodied in the outputs of sector s_p (aggregated across producing regions) *directly supplied to final demand* in r_f in year t.	$E^{\mathrm{CBA}_{FD}}_{e,(r_f,s_p)}(t) = \sum_{r_p\in\mathrm{Reg}} E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p),r_f}(t)$ $= D^{cba}_{e,(r_f,s_p)}(t)$

Enacting metric	Definition	Equation
$E^{\text{CBA}_{TD}}_{e,(r_c,s_p)}(t)$	Quantity of environmental impact category e embodied in the outputs of sector s_p (aggregated across producing regions) *directly supplied to total demand* (final + intermediate) in r_c in year t.	$E^{\text{CBA}_{TD}}_{e,(r_c,s_p)}(t) = \sum_{r_p \in \text{Reg}} E^{\text{CBA}_{TD}}_{(e,r_p,s_p),r_c}(t)$ $= D^{cba}_{e,(r_c,s_p)}(t) + \sum_{r_p \in \text{Reg}} M_{e,(r_p,s_p)}(t) \sum_{s_c \in \text{Sec}} Z_{(r_p,s_p),(r_c,s_c)}(t)$
$E^{\text{PBA}}_{e,(r_p,s_p)}(t)$	Direct environmental impact category e generated by producing sector-region pair (r_p, s_p) in year t.	$E^{\text{PBA}}_{e,(r_p,s_p)}(t) = S_{e,(r_p,s_p)}(t)\; x_{(r_p,s_p)}(t) = F_{e,(r_p,s_p)}(t) = D^{pba}_{e,(r_p,s_p)}(t)$
$GVA_{(r_p,s_p)}(t)$	Direct GVA generated by producing sector-region pair (r_p, s_p) in year t.	$GVA_{(r_p,s_p)}(t) = \sum_{v \in VA} S_{v,(r_p,s_p)}(t)\; x_{(r_p,s_p)}(t) \quad = \sum_{v \in VA} F_{v,(r_p,s_p)}(t)$ $= \sum_{v \in VA} D^{pba}_{v,(r_p,s_p)}(t)$

FD = final demand; TD = total demand (final + intermediate); GVA = gross value added; PBA = production-based accounting; CBA_{FD} = consumption-based accounting of final demand; CBA_{TD} = consumption-based accounting of total demand.

3.2.2 L_2 allocation equations by functional unit

Using the enacting metrics defined above, Table 7 reports the allocation equations covered by *UNCASExt* at L_2, organized by functional unit. *UNCASExt* supports both (i) a two-step formulation that first allocates shares of carrying capacities at L_1 and then distributes them across sectors of the L_1 region, and (ii) a one-step formulation that allocates directly from the global level to L_2 sectors.

UNCASExt proposes new L_2 utilitarian allocation methods to address CBA_{TD} accounting system boundaries, building on previous work by [Oosterhoff et al., 2023, Lenzen et al., 2007, Gopalakrishnan et al., 2021, Gopalakrishnan, 2022]. These methods define adjusted enacting metrics and propagation rules that compute aSoCCs consistent with accounting system boundaries targeting the Scope 1–2–3 impacts of a sector's total demand/output (i.e., CBA_{TD}). The consumption-anchored allocation method UT(FDa) attributes to an upstream sector, via the Ghosh inverse [Lenzen and Murray, 2010], the final demand it enables through direct and indirect downstream supply chains. Therefore, it extends UT(FD) from final demand-only interpretations to CBA_{TD} settings, where intermediate demand circulation must be accounted for. It is an update of the approach in [Oosterhoff et al., 2023]. The production-anchored allocation method UT(GVAa) attributes to a downstream sector the value added it mobilizes directly and indirectly along its upstream supply chain [Lenzen et al., 2007, Gopalakrishnan et al., 2021, Gopalakrishnan, 2022]. Therefore, it extends UT(GVA) beyond Scope 1 (direct value added) to a Scope 1–2–3 interpretation consistent with total demand/output under CBA_{TD} system boundaries.

The UT(FDa) and UT(GVAa) equations are reported in Section 4; Table 7 references these equations for the corresponding L_2 functional units.

Table 7: L_2 allocation methods covered by UNCASExt, by functional unit and accounting system boundaries

SP(EM)	L_1 weighting	Definition	Equation	Original UNCASE
FU $L_{2.a.a}$ (CBA_{FD}): Total production of goods and services by sector s_p in producing region(s) r_p directly supplied to final demand worldwide in year t				
$\boldsymbol{UT}\mathbf{(FD)}_{(r_p,s_p)}(t)$	✓	Consumption-anchored utility: allocate the $aSoCC^{L_{1.a}}_{r_f}(t)$ of all regions r_f across producers (r_p, s_p) directly supplying FD *worldwide* in year t.	$\sum_{r_f \in Reg} aSoCC^{L_{1.a}}_{r_f}(t)\; \frac{FD_{(r_p,s_p),r_f}(t)}{FD_{r_f}(t)}$	✗
$\boldsymbol{UT}\mathbf{(FD)}_{(r_p,s_p)}(t)$	✗	Consumption-anchored utility: one-step global to producers (r_p, s_p) directly supplying FD *worldwide* in year t.	$\frac{FD_{(r_p,s_p)}(t)}{\sum_{r_f \in Reg} FD_{r_f}(t)}$	✗

SP(EM)	L_1 **weighting**	**Definition**	**Equation**	**Original UNCASE**
$\boldsymbol{AR(E^{\mathrm{CBA}_{FD}})}_{(e,r_p,s_p)}(t^{ar})$	✓	Acquired rights: allocate the $aSoCC^{L_{1.a}}_{r_f}(t)$ of all regions r_f across producers (r_p, s_p) directly supplying FD *worldwide* via CBA_{FD} impacts in reference year t^{ar}.	$\displaystyle\sum_{r_f \in Reg} aSoCC^{L_{1.a}}_{r_f}(t)\ \frac{E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p),r_f}(t^{ar})}{E^{\mathrm{CBA}_{FD}}_{e,r_f}(t^{ar})}$	✗
$\boldsymbol{AR(E^{\mathrm{CBA}_{FD}})}_{(e,r_p,s_p)}(t^{ar})$	✗	Acquired rights: one-step global to producers (r_p, s_p) directly supplying FD *worldwide* via CBA_{FD} impacts in reference year t^{ar}.	$\dfrac{E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p)}(t^{ar})}{\sum_{r_f \in Reg} E^{\mathrm{CBA}_{FD}}_{e,r_f}(t^{ar})}$	✗

SP(EM)	L_1 **weighting**	**Definition**	**Equation**	**Original UNCASE**
FU $L_{2.a.b}$ (CBA$_{\mathrm{TD}}$)*: Total production of goods and services by sector* s_p *in producing region(s)* r_p *in year* t				
$\boldsymbol{UT}\textbf{(FDa)}_{(r_p,s_p)}(t)$	✓	Adjusted consumption-anchored utility: allocate via the Ghosh inverse the $aSoCC^{L_{1.a}}_{r_f}(t)$ of all regions r_f across producers (r_p, s_p) directly and indirectly supplying FD *worldwide* in year t.	$\sum_{r_f \in \mathrm{Reg}} aSoCC^{L_{1.a}}_{r_f}(t)\ \frac{x_{(r_p,s_p)}(t)\ \kappa_{(r_p,s_p),\,r_f}(t)}{FD_{r_f}(t)}$ See Section 4, Eq. (30)	✗
$\boldsymbol{UT}\textbf{(GVAa)}_{(r_p,s_p)}(t)$	✓	Adjusted production-anchored utility: allocate via the Leontief inverse the $aSoCC^{L_{1.b}}_{r_p}$ of all regions r_u across producers (r_p, s_p) directly and indirectly mobilizing GVA *worldwide* in year t.	$\sum_{r_u \in \mathrm{Reg}} aSoCC^{L_{1.b}}_{r_u}(t)\ \frac{x_{(r_p,s_p)}(t) \sum_{s_u \in \mathrm{Sec}} \omega_{(r_u,s_u),(r_p,s_p)}(t)}{GVA_{r_u}(t)}$ See Section 4, Eq. (43)	✗
$\boldsymbol{UT}\textbf{(TD)}_{(r_p,s_p)}(t)$	✗	One-step global to producers (r_p, s_p) direct and indirect FD/GVA utility in year t (one-step: UT(FDa)=UT(GVAa, see Section 4.4).	$\frac{x_{(r_p,s_p)}(t)}{\sum_{r_f \in Reg} FD_{r_f}(t)}$ See Section 4, Eqs. (31) & (44)	✗
$\boldsymbol{AR}(\boldsymbol{E}^{\mathbf{CBA}_{TD}})_{e,(r_p,s_p)}(t^{ar})$	✗	Acquired rights: one-step global to producers (r_p, s_p) via CBA$_{TD}$ impacts in reference year t^{ar}.	$\frac{E^{\mathrm{CBA}_{TD}}_{e,(r_p,s_p)}(t^{ar})}{\sum_{r \in Reg} E^{\mathrm{CBA}_{FD}}_{e,r}(t^{ar})}$	✗
FU $L_{2.a.c}$ *(PBA): Total production of goods and services by sector* s_p *in producing region(s)* r_p *in year* t				
$\boldsymbol{UT}\textbf{(GVA)}_{(r_p,s_p)}(t)$	✓	Production-anchored utility: allocate $aSoCC^{L_{1.b}}_{r_p}(t)$ across producers (r_p, s_p) directly mobilizing GVA in r_p in year t.	$aSoCC^{L_{1.b}}_{r_p}(t)\ \frac{GVA_{(r_p,s_p)}(t)}{GVA_{r_p}(t)}$	✓
$\boldsymbol{UT}\textbf{(GVA)}_{(r_p,s_p)}(t)$	✗	Production-anchored utility: one-step global to outputs of (r_p, s_p) directly mobilizing GVA in r_p in year t.	$\frac{GVA_{(r_p,s_p)}(t)}{\sum_{r'_p \in Reg} GVA_{r'_p}(t)}$	✓
$\boldsymbol{AR}(\boldsymbol{E}^{\mathbf{PBA}})_{e,(r_p,s_p)}(t^{ar})$	✓	Acquired rights: allocate $aSoCC^{L_{1.b}}_{r_p}(t)$ across producers (r_p, s_p) directly mobilizing GVA in r_p via *PBA* impacts in reference year t^{ar}.	$aSoCC^{L_{1.b}}_{r_p}(t)\ \frac{E^{\mathrm{PBA}}_{e,(r_p,s_p)}(t^{ar})}{E^{\mathrm{PBA}}_{e,r_p}(t^{ar})}$	✗
$\boldsymbol{AR}(\boldsymbol{E}^{\mathbf{PBA}})_{e,(r_p,s_p)}(t^{ar})$	✗	Acquired rights: one-step global to producers (r_p, s_p) directly mobilizing GVA in r_p via *PBA* impacts in reference year t^{ar}.	$\frac{E^{\mathrm{PBA}}_{e,(r_p,s_p)}(t^{ar})}{\sum_{r'_p \in Reg} E^{\mathrm{PBA}}_{e,r'_p}(t^{ar})}$	✗
FU $L_{2.b.a}$ (CBA$_{\mathrm{FD}}$)*: Total production of goods and services by sector* s_p *in producing region(s)* r_p *directly supplied to final demand in region(s)* r_f *in year* t				
$\boldsymbol{UT}\textbf{(FD)}_{(r_p,s_p),r_f}(t)$	✓	Consumption-anchored utility: allocate $aSoCC^{L_{1.a}}_{r_f}(t)$ across producers (r_p, s_p) directly supplying FD in r_f in year t.	$aSoCC^{L_{1.a}}_{r_f}(t)\ \frac{FD_{(r_p,s_p),r_f}(t)}{FD_{r_f}(t)}$	✗
$\boldsymbol{UT}\textbf{(FD)}_{(r_p,s_p),r_f}(t)$	✗	Consumption-anchored utility: one-step global to producers (r_p, s_p) directly supplying FD in r_f in year t.	$\frac{FD_{(r_p,s_p),r_f}(t)}{\sum_{r_f \in Reg} FD_{r_f}(t)}$	✗
$\boldsymbol{AR}(\boldsymbol{E}^{\mathbf{CBA}_{FD}})_{(e,r_p,s_p),r_f}(t^{ar})$	✓	Acquired rights: allocate $aSoCC^{L_{1.a}}_{r_f}(t)$ across producers (r_p, s_p) directly supplying FD in r_f via CBA_{FD} impacts in reference year t^{ar}.	$aSoCC^{L_{1.a}}_{r_f}(t)\ \frac{E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p),r_f}(t^{ar})}{E^{\mathrm{CBA}_{FD}}_{e,r_f}(t^{ar})}$	✗
$\boldsymbol{AR}(\boldsymbol{E}^{\mathbf{CBA}_{FD}})_{(e,r_p,s_p),r_f}(t^{ar})$	✗	Acquired rights: one-step global to producers (r_p, s_p) directly supplying FD in r_f via CBA_{FD} impacts in reference year t^{ar}.	$\frac{E^{\mathrm{CBA}_{FD}}_{(e,r_p,s_p),r_f}(t^{ar})}{\sum_{r'_f \in Reg} E^{\mathrm{CBA}_{FD}}_{e,r'_f}(t^{ar})}$	✗

SP(EM)	L_1 weighting	Definition	Equation	Original UNCASE
FU $L_{2.b.b}$ (CBA$_{\mathrm{TD}}$): Total production of goods and services by sector s_p in producing region(s) r_p directly supplied to total demand in region(s) r_c in year t				
$\boldsymbol{UT}\mathbf{(FDa)}_{(r_p,s_p),r_c}(t)$	✓	Adjusted consumption-anchored utility: allocate the $aSoCC^{L_{1.a}}_{r_f}(t)$ of all regions r_f across producers (r_p, s_p) directly supplying TD in r_c via the resulting direct and indirect contributions to FD *worldwide* (Ghosh inverse propagation) in year t.	$\sum_{r_f \in \mathrm{Reg}} aSoCC^{L_{1.a}}_{r_f}(t) \frac{x_{(r_p,s_p),r_c}(t)\ \kappa_{(r_c,r_p,s_p),\, r_f}(t)}{FD_{r_f}(t)}$ See Section 4, Eq. (32)	✗
$\boldsymbol{UT}\mathbf{(GVAa)}_{(r_p,s_p),r_c}(t)$	✓	Adjusted production-anchored utility: allocate via the $aSoCC^{L_{1.b}}_{r_p}$ of all regions r_u across producers (r_p, s_p) directly supplying TD in r_c via the GVA mobilized directly and indirectly *worldwide* (Leontief inverse propagation) in year t.	$\sum_{r_u \in \mathrm{Reg}} aSoCC^{L_{1.b}}_{r_u}(t) \frac{x_{(r_p,s_p),r_c}(t)\ \sum_{s_u \in \mathrm{Sec}} \omega_{(r_u,s_u),(r_p,s_p)}(t)}{GVA_{r_u}(t)}$ See Section 4, Eq. (45)	✗
$\boldsymbol{UT}\mathbf{(TD)}_{(r_p,s_p),r_c}(t)$	✗	One-step global to producers (r_p, s_p) directly supplying TD in r_c via the resulting direct and indirect FD/GVA *worldwide* utility in year t (one-step: UT(FDa)=UT(GVAa, see Section 4.4).	$\frac{x_{(r_p,s_p),r_c}(t)}{\sum_{r_f \in Reg} FD_{r_f}(t)}$ See Section 4, Eqs. (33) & (46)	✗
$\boldsymbol{AR}(\boldsymbol{E}^{\mathbf{CBA}_{\boldsymbol{TD}}})_{(e,r_p,s_p),r_c}(t^{ar})$	✗	Acquired rights: one-step global to producers (r_p, s_p) directly supplying TD in r_c via CBA$_{TD}$ impacts in reference year t^{ar}.	$\frac{E^{\mathrm{CBA}_{TD}}_{(e,r_p,s_p),r_c}(t^{ar})}{\sum_{r \in Reg} E^{\mathrm{CBA}_{FD}}_{e,r}(t^{ar})}$	✗
FU $L_{2.c.a}$ (CBA$_{\mathrm{FD}}$): Final demand in region(s) r_f in year t of goods and services produced by sector s_p				
$\boldsymbol{UT}\mathbf{(FD)}_{(r_f,s_p)}(t)$	✓	Consumption-anchored utility: allocate $aSoCC^{L_{1.a}}_{r_f}(t)$ across producers (s_p) – i.e., *worldwide production* – directly supplying FD in r_f in year t.	$aSoCC^{L_{1.a}}_{r_f}(t) \frac{FD_{(r_f,s_p)}(t)}{FD_{r_f}(t)}$	✓
$\boldsymbol{UT}\mathbf{(FD)}_{(r_f,s_p)}(t)$	✗	Consumption-anchored utility: one-step global to producers (s_p) – i.e., *worldwide production* – directly supplying FD in r_f in year t.	$\frac{FD_{(r_f,s_p)}(t)}{\sum_{r_f \in Reg} FD_{r_f}(t)}$	✓
$\boldsymbol{AR}(\boldsymbol{E}^{\mathbf{CBA}_{\boldsymbol{FD}}})_{e,(r_f,s_p)}(t^{ar})$	✓	Acquired rights: allocate $aSoCC^{L_{1.a}}_{r_f}(t)$ across producers (s_p) – i.e., *worldwide production* –directly supplying FD in r_f via CBA_{FD} impacts in reference year t^{ar}.	$aSoCC^{L_{1.a}}_{r_f}(t) \frac{E^{\mathrm{CBA}_{FD}}_{e,(r_f,s_p)}(t^{ar})}{E^{\mathrm{CBA}_{FD}}_{e,r_f}(t^{ar})}$	✓
$\boldsymbol{AR}(\boldsymbol{E}^{\mathbf{CBA}_{\boldsymbol{FD}}})_{e,(r_f,s_p)}(t^{ar})$	✗	Acquired rights: one-step global to producers (s_p) – i.e., *worldwide production* – directly supplying FD in r_f via CBA_{FD} impacts in reference year t^{ar}.	$\frac{E^{\mathrm{CBA}_{FD}}_{e,(r_f,s_p)}(t^{ar})}{\sum_{r'_f \in Reg} E^{\mathrm{CBA}_{FD}}_{e,r'_f}(t^{ar})}$	✓
FU $L_{2.c.b}$ (CBA$_{\mathrm{TD}}$): Total demand in region(s) r_c in year t of goods and services produced by sector s_p				
$\boldsymbol{UT}\mathbf{(FDa)}_{(r_c,s_p)}(t)$	✓	Adjusted consumption-anchored utility: allocate the $aSoCC^{L_{1.a}}_{r_f}(t)$ of all regions r_f across producers (s_p) – i.e., *worldwide production* – directly supplying TD in r_c via the resulting direct and indirect contributions to FD *worldwide* (Ghosh inverse propagation) in year t.	$\sum_{r_f \in \mathrm{Reg}} aSoCC^{L_{1.a}}_{r_f}(t) \frac{\sum_{r_p \in \mathrm{Reg}} x_{(r_p,s_p),r_c}(t)\ \kappa_{(r_c,s_p),\, r_f}(t)}{FD_{r_f}(t)}$ See Section 4, Eq. (35)	✗
$\boldsymbol{UT}\mathbf{(GVAa)}_{(r_c,s_p)}(t)$	✓	Adjusted production-anchored utility: allocate via the $aSoCC^{L_{1.b}}_{r_p}$ of all regions r_u across producers (s_p) – i.e., *worldwide production* – directly supplying TD in r_c via the GVA mobilized directly and indirectly *worldwide* (Leontief inverse propagation) in year t.	$\sum_{r_u \in \mathrm{Reg}} aSoCC^{L_{1.b}}_{r_u}(t) \frac{\sum_{r_p \in \mathrm{Reg}} x_{(r_p,s_p),r_c}(t)\ \sum_{s_u \in \mathrm{Sec}} \omega_{(r_u,s_u),(r_p,s_p)}(t)}{GVA_{r_u}(t)}$ See Section 4, Eq. (47)	✗

SP(EM)	L_1 weighting	Definition	Equation	Original UNCASE
$\boldsymbol{UT}\mathbf{(TD)}_{(r_c,s_p)}(t)$	✗	One-step global to producers (s_p) – i.e., *worldwide production* – directly supplying TD in r_c via the resulting direct and indirect FD/GVA *worldwide* utility in year t (one-step: UT(FDa)=UT(GVAa, see Section 4.4).	$\frac{\sum_{r_p \in \text{Reg}} x_{(r_p,s_p),r_c}(t)}{\sum_{r_f \in Reg} FD_{r_f}(t)}$ See Section 4, Eqs. (36) & (48)	✗
$\boldsymbol{AR}(\boldsymbol{E}^{\mathbf{CBA}_{TD}})_{e,(r_c,s_p)}(t^{ar})$	✗	Acquired rights: one-step global to producers (s_p) – i.e., *worldwide production* – directly supplying TD in r_c via CBA$_{TD}$ impacts in reference year t^{ar}.	$\frac{E^{\text{CBA}_{TD}}_{e,(r_c,s_p)}(t^{ar})}{\sum_{r \in Reg} E^{\text{CBA}_{FD}}_{e,r}(t^{ar})}$	✗

SP = sharing principle; EM = enacting metric; FU = functional unit; aSoCC = allocated share of carrying capacity; FD = final demand; FDa = adjusted final demand (via the Ghosh inverse); TD = total demand (final + intermediate); GVA = gross value added; GVAa = adjusted gross value added (via the Leontief inverse); PBA = production-based accounting; CBA$_{\text{FD}}$ = consumption-based accounting of final demand; CBA$_{\text{TD}}$ = consumption-based accounting of total demand; UT = utilitarian; AR = acquired rights (grandfathering).

4 Definition of new L_2: utilitarian allocation methods for CBA_{TD} system boundaries

4.1 Motivations

Rationale and ethical anchoring. The adjusted utilitarian enacting metrics UT(FDa) and UT(GVAa) are introduced to address AESA studies in which the ASR numerator (Phase A), i.e., the estimation of environmental burdens, is formulated under CBA_{TD} system boundaries. In such cases, the denominator (Phase B), i.e., the allocated carrying capacities, should represent the same accounting system boundaries and demand perimeter, namely consumption-based accounting (CBA) and total demand (TD), where total demand includes both final demand (BtoC) and intermediate demand (BtoB). This is not ensured by direct utilitarian allocation methods such as UT(FD), which is limited to final demand (FD), or UT(GVA), which reflects only the value added directly generated by the studied activity and therefore follows a production-based accounting (PBA) logic.

UT(FDa) and UT(GVAa) both target CBA_{TD} system boundaries, but through two allocation paths with distinct ethical anchoring. UT(FDa)-based allocation follows a consumption-anchored path, in which shares of carrying capacities are allocated according to consumption. UT(GVAa)-based allocation follows a production-anchored path, in which shares of carrying capacities are allocated according to production.

On the consumption-anchored side, UT(FDa) extends FD as a proxy for utility by measuring how the studied activity's output contributes directly and indirectly to final demand. This includes output sold directly to final demand as well as output sold to downstream sectors, which may then pass through one or several subsequent production stages before contributing to final demand. UT(FDa) therefore propagates the studied activity's output downstream to identify the final demand enabled by this output, using the Ghosh inverse [Lenzen and Murray, 2010]. It is conceptually close to the total demand approach proposed by [Oosterhoff et al., 2023], but provides greater flexibility by enabling filtering according to the country where the studied activity's outputs are first sold, either to final demand or to a downstream sector.

On the production-anchored side, UT(GVAa) extends GVA as a proxy for utility by applying the reverse logic: it measures the value added created upstream of the studied activity to enable the production of its total output. This includes the value added generated by the studied activity itself as well as the value added generated by upstream sectors that supply, through one or several production stages, the inputs required for its production. This addresses the limitation of UT(GVA), which only reflects value added generated directly by the studied activity (Scope 1), and may therefore lead to underallocation when the ASR numerator (Phase A) includes upstream burdens [Balanza et al., 2025]. UT(GVAa) is consistent with value added attribution approaches that trace value added through supply chains according to upstream production dependencies [Lenzen et al., 2007, Gopalakrishnan et al., 2021, Gopalakrishnan, 2022].

Additivity. At the world level, direct allocation methods such as UT(FD) and UT(GVA) allocate the full carrying capacity once across all activities, so their allocated shares sum to one. By contrast, UT(FDa) and UT(GVAa) start from these direct shares and propagate them through MRIO trade structures to represent total demand/total output under CBA accounting system boundaries. This propagation creates overlapping allocation perimeters across sectors, since a share initially assigned to final demand or value added can also be assigned to the upstream or downstream activities that enable it. Consequently, the sum of allocated shares across all activities may exceed one [Oosterhoff et al., 2023, Bjørn et al., 2023].

This non-additivity is expected under CBA_{TD} system boundaries. It reflects the fact that overlapping system boundaries must be represented consistently in both the ASR numerator (Phase A), i.e., the estimation of environmental burdens, and the denominator (Phase B), i.e., the allocated carrying capacities. In Phase A, the estimation of environmental burdens of a studied activity may include upstream burdens that also fall within the system boundaries of other activities through Scope 2 or Scope 3 relationships. Phase B should mirror this structure when carrying capacity are allocated to activities whose total demand/total output is represented on a CBA basis.

Schematic representation. Figure 2 provides a schematic MRIO system to illustrate the direct and indirect supply chain relationships between region–sector pairs. In a direct UT(GVA) logic, only the local value added v of the studied node is taken into account, while upstream v is ignored even though it is mobilized through Z to produce the studied output. In a direct UT(FD) logic, only deliveries to Y are taken into account, so intermediate deliveries via Z are ignored unless an adjusted propagation mechanism is introduced.

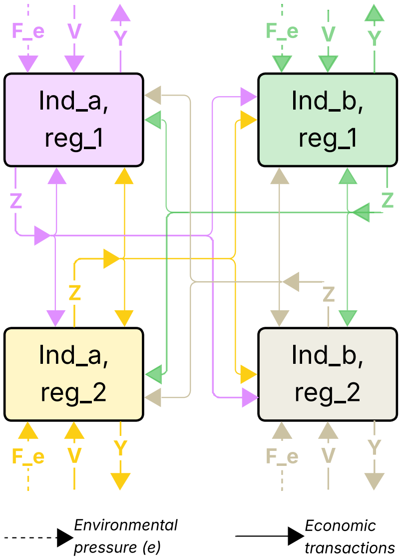


Figure 2: Mock two-region, two-sector MRIO system. Direct UT(GVA) considers only direct value added v (scope 1, production-based accounting), and therefore omits the upstream value added V embodied in the intermediate inputs Z that are mobilized to produce the studied output. Direct UT(FD) considers only deliveries to final demand Y, and therefore omits the utility that is generated downstream when the studied producing pair (r_p, s_p) supplies intermediate outputs through Z to sectors that later (directly or indirectly) deliver to final demand.

Summary comparison. Table 8 summarizes the two adjusted utilitarian allocation methods introduced by *UNCASExt* and contrasts them with their direct counterparts. The methods are described in detail in the next subsections.

Table 8: Summary comparison of the two adjusted utilitarian allocation methods in MRIO (UT(FDa) versus UT(GVAa)).

Method	**Anchoring**	**Direct EM**	**Adjustment goal**	**Propagation operator and interpretation**
UT(FDa)	Consumption-anchored: upstream attribution of downstream FD deliveries	Direct FD deliveries (Y, by destination region r_f)	Attribute to an upstream producer (r_p, s_p) the FD it enables via direct and indirect downstream supply chains	Downstream propagation using the Ghosh inverse $G(t) = \big(I - B(t)\big)^{-1}$, allocating by recursive inheritance of downstream sectors' direct and indirect FD contributions (downstream scope 3).
UT(GVAa)	Production-anchored: downstream attribution of upstream GVA creation	Direct GVA creation$\sum_{v \in VA} F_{v,(r,s)}(t)$ (by producing pair (r, s))	Attribute to a downstream producer (r_p, s_p) the GVA it mobilizes directly and indirectly along its upstream supply chain	Upstream propagation using the Leontief inverse $L(t) = \big(I - A(t)\big)^{-1}$, allocating inherited supplier GVA required directly and indirectly to produce total output (scope 2 and upstream scope 3).

UT = utilitarian; EM = enacting metric; GVA = gross value added; FD = final demand; FDa = adjusted final demand (via the Ghosh inverse); GVAa = adjusted gross value added (via the Leontief inverse).

Future research. In the current *UNCASExt* implementation, the L_2 utility proxy is purely monetary. Consequently, one unit of economic output is valued equally across all sectors, and normative differentiation can only enter at L_1 by weighting countries' allocated shares according to the selected sharing principle (i.e., differentiating across countries rather than across types of goods and services).

For future research, *within-*L_2 weighting schemes could differentiate utility across final demand categories, for instance by assigning higher weights to categories that satisfy essential needs. This is the rationale of sufficientarian approaches [Paulillo and Sanyé-Mengual, 2024, Bjørn et al., 2026]. Under CBA$_{FD}$ accounting system boundaries, such weights can be applied directly at the final demand stage (e.g., [Kromand et al., 2025]). Extending sufficientarian weighting to CBA$_{TD}$ accounting system boundaries would require propagating category weights through upstream supply chains, for example via the proposed UT(FDa) method, so that utility is valued uniformly *within* each final demand category but not *across* categories, and a sector's contribution depends on both (i) the final demand categories it ultimately enables and (ii) the associated category-specific weights.

Operationalizing this extension would require (i) sufficientarian allocation methods weighting final demand categories that scale across many countries (potentially global coverage given multi-regional supply chains) and (ii) a consistent mapping between the resulting essential need categories and MRIOs sector/product classifications, so that each sector's direct and indirect contribution to each weighted final demand category could be traced through supply chains. Recent proposals such as [Kromand et al., 2025] are promising starting points, but would need to be generalized beyond single-country applications and harmonized with MRIO classifications to be operational in this context.

4.2 UT(FDa): upstream attribution of downstream final demand responsibility

Motivation. Utilitarian allocation methods that rely solely on *direct* final demand systematically underallocate utility to intermediate-supplying sectors. To address this limit, [Oosterhoff et al., 2023] proposed an allocation method that attributes a share of final demand to a producing pair (r_p, s_p) based on its estimated *direct and indirect* contributions to final demand via downstream sectors. Hereafter, this method is denoted UT(FD_{B2M}) for clarity, following the title of [Oosterhoff et al., 2023].

This section (i) summarizes UT(FD_{B2M}) and its limitations for the functional units covered by *UNCASExt*, and (ii) introduces a new downstream propagation formulation, denoted UT(FDa), that is compatible with all *UNCASExt* L_2 functional units described in Subsection 1.2. The new formulation relies on the Ghosh inverse [Lenzen and Murray, 2010] to perform downstream indirect attribution of final demand and follows `pymrio` notation conventions [Stadler, 2021].

4.2.1 Exclusion of [Oosterhoff et al., 2023] adjusted final demand approach via the Leontief inverse

Principle. UT(FD_{B2M}) [Oosterhoff et al., 2023] is anchored in final demand by region r_f. It proceeds in two conceptual layers. First, within each r_f, it identifies which producing pairs directly supply final demand. Second, it reallocates these direct final demand delivery shares to upstream producers using an allocation kernel derived from the Leontief inverse. Therefore, although the objective is to quantify downstream contributions to final demand (direct and indirect via downstream sectors), the propagation operator is built from the input-driven Leontief system, which is naturally suited to upstream tracing.

Notation and indices. a sector-region pair is denoted (r, s). We distinguish (r_p, s_p) for the studied producing pair receiving an allocated share, (r_d, s_d) for a producing pair that directly supplies final demand in r_f, and r_f for the region where final demand occurs.

The MRIO matrices are the technical coefficients matrix $A(t)$, the Leontief inverse $L(t) = (I - A(t))^{-1}$, and the final demand matrix $Y(t)$.

4.2.1.1 Step 1: Direct final demand delivery shares Following Eq. (1) in [Oosterhoff et al., 2023], the direct share of final demand in region r_f supplied by producing pair (r_d, s_d) is

$$FR_{(r_d,s_d),\,r_f}(t) = \frac{\sum_{y\in FD} Y_{(r_d,s_d),(r_f,y)}(t)}{\sum_{r'_d\in\text{Reg}}\sum_{s'_d\in\text{Sec}}\sum_{y\in FD} Y_{(r'_d,s'_d),(r_f,y)}(t)}, \qquad \sum_{r_d\in\text{Reg}}\sum_{s_d\in\text{Sec}} FR_{(r_d,s_d),\,r_f}(t) = 1. \tag{14}$$

4.2.1.2 Step 2: Propagation basis (Leontief inverse). Following Eq. (2) in [Oosterhoff et al., 2023],

$$L(t) = \left(I - A(t)\right)^{-1}, \qquad L_{(r_p,s_p),(r_d,s_d)}(t) = \left[\left(I - A(t)\right)^{-1}\right]_{(r_p,s_p),(r_d,s_d)}. \tag{15}$$

$L_{(r_p,s_p),(r_d,s_d)}(t)$ gives the total (direct and indirect) output from (r_p, s_p) required per unit of output of (r_d, s_d). By construction, $L(t)$ propagates *upstream* requirements, from downstream activities to their upstream suppliers.

4.2.1.3 Step 3: Leontief-based reallocation using the scaling matrix $S(t)$. UT(FD_{B2M}) reallocates the direct shares in Eq. (14) to upstream producers by combining $FR(t)$ with column-normalized coefficients built from a scaling matrix $S(t)$ derived from the Leontief inverse (Eqs. (3) to (6) in [Oosterhoff et al., 2023]). Using the notation of [Oosterhoff et al., 2023], the allocation has the generic structure

$$UT(FD_{B2M})_{(r_p,s_p),\,r_f}(t) = \sum_{r_d\in\text{Reg}}\sum_{s_d\in\text{Sec}} FR_{(r_d,s_d),\,r_f}(t)\,\frac{S_{(r_p,s_p),(r_d,s_d)}(t)}{\widehat{S}_{(r_d,s_d),(r_d,s_d)}(t)}, \tag{16}$$

where $\widehat{S}(t)$ denotes the diagonal part used for the column normalization. In [Oosterhoff et al., 2023], $S(t)$ is obtained by combining a diagonal term $\widehat{S}(t)$ with an off-diagonal term $\widetilde{S}(t)$, where $\widetilde{S}(t)$ is constructed from $L(t)$ using an auxiliary factor $f(t)$ (Eqs. (4) to (5) in [Oosterhoff et al., 2023]) to approximate a “full economic output” interpretation (Scope 1, Scope 2, Scope 3) while operating within an upstream Leontief framework.

4.2.1.4 Step 4: Optional L_1 weighting over final demand regions.

The final step in [Oosterhoff et al., 2023] applies an equal-per-capita weighting over r_f:

$$UT(FD_{B2M})_{(r_p,s_p)}(t) = \sum_{r_f \in \text{Reg}} EG(Pop)_{r_f}(t)\, UT(FD_{B2M})_{(r_p,s_p),r_f}(t). \tag{17}$$

4.2.1.5 Limitations for UNCASExt.

The target quantity in UT(FD_{B2M}) is the *direct and indirect contribution of a producing pair* (r_p, s_p) *to final demand*, where indirect contribution arises because (r_p, s_p) sells outputs to downstream sectors that, through further downstream steps, ultimately deliver to final demand. This is therefore a *downstream attribution* question.

However, the construction in [Oosterhoff et al., 2023] starts from the downstream endpoint (direct final demand delivery shares FR) and redistributes them upstream using an operator derived from the Leontief inverse. This has a practical implication: UT(FD_{B2M}) is naturally expressed for the functional unit $L_{2.a.b}$ (CBA accounting for a producing pair (r_p, s_p) considering all its outputs), but it does not directly support functional units that require conditioning on the *first buyer region* r_c, namely $L_{2.b.b}$ and, by aggregation, $L_{2.c.b}$ (see Table 2). These functional units require filtering the studied outputs by where they are first directly sold (to total demand in r_c) *before* tracing downstream contributions to final demand. With a redistribution that begins at final demand and moves upstream, that first-buyer conditioning is not available as an intrinsic part of the propagation operator. This limits the applicability of the method for case studies where the intended utility perimeter is consumption by a given region's total demand rather than the production of a given sector-region pair. For example, this is the functional unit of the original UNCASE case study [Puig-Samper et al., 2025]. Its implementation of UT(FD_{B2M}), therefore, leads to a functional unit focusing on French electricity produced in France wherever consumed ($L_{2.a.b}$) rather than the intended one of electricity consumed by the French total demand ($L_{2.c.b}$).

4.2.2 UT(FDa): upstream attribution of downstream final demand responsibility via the Ghosh inverse

Principle. UT(FDa) allocates to each producing pair (r_p, s_p) a share of final demand that reflects both: (i) its *direct* deliveries to final demand, and (ii) its *direct and indirect* contributions to downstream producing sectors that ultimately deliver to final demand.

A key requirement in *UNCASExt* is compatibility with functional units that condition the studied activity on the *first-sale* region r_c (i.e., where outputs are first sold to total demand: intermediate and/or final demand). This requires a *downstream* tracing operator in which first-sale information enters explicitly through observed first-sale flows in Z and Y.

Functional units. The UT(FDa) allocation method is compatible with the three *UNCASExt* L_2 functional units (Table 2):

- $L_{2.a.b}$: Total production of goods and services by sector s_p in producing region(s) r_p in year t.
- $L_{2.b.b}$: Total production of goods and services by sector s_p in producing region(s) r_p directly supplied to total demand in region(s) r_c in year t.
- $L_{2.c.b}$: Total demand in region(s) r_c in year t of goods and services produced by sector s_p.

Notation and indices. A producing sector–region pair is denoted (r, s). We distinguish (r_p, s_p) for the producing pair whose UT(FDa) is reported, (r_c, s_c) for a downstream sector in the first-sale region r_c, and r_f for the region where final demand occurs.

Let $Z(t)$ be the transaction matrix, $Y(t)$ the final demand matrix, $x(t)$ the gross output vector, and u the vector of ones. Following `pymrio` notations [Stadler, 2021, Lenzen and Murray, 2010], define the input share matrix and the Ghosh inverse by

$$B(t) = \widehat{x}(t)^{-1} Z(t), \qquad G(t) = \big(I - B(t)\big)^{-1}. \tag{18}$$

4.2.2.1 Step 1: Direct final demand shares by destination region. For each producing pair (r, s) and consuming (final demand) region r_f, define the *direct* share of output of (r, s) that is delivered to final demand in r_f:

$$\theta_{(r,s),r_f}(t) = \frac{\sum\limits_{y \in FD} Y_{(r,s),(r_f,y)}(t)}{x_{(r,s)}(t)}. \tag{19}$$

Collect these coefficients into the matrix $\Theta(t)$ with entries $\theta_{(r,s),r_f}(t)$. By the output identity $x(t) = Z(t)\,u + Y(t)\,u$, one has

$$\sum_{r_f \in \mathrm{Reg}} \theta_{(r,s),r_f}(t) = \frac{\sum_{r_f \in \mathrm{Reg}} \sum_{y \in FD} Y_{(r,s),(r_f,y)}(t)}{x_{(r,s)}(t)} \leq 1,$$

with equality if the producer sells only to final demand (no intermediate sales).

4.2.2.2 Step 2: Ultimate destination shares via Ghosh propagation. Define the matrix of *ultimate* destination shares by

$$\Pi(t) = G(t)\,\Theta(t). \tag{20}$$

Equivalently, elementwise,

$$\pi_{(r,s),r_f}(t) = \sum_{r' \in \mathrm{Reg}} \sum_{s' \in \mathrm{Sec}} G_{(r,s),(r',s')}(t)\;\theta_{(r',s'),r_f}(t), \qquad \sum_{r_f \in \mathrm{Reg}} \pi_{(r,s),r_f}(t) = 1. \tag{21}$$

$\pi_{(r,s),r_f}(t)$ is interpretable as: *the share of one unit of output of (r, s) that ultimately ends in final demand in consuming region r_f, accounting for direct and indirect downstream pathways.*

4.2.2.3 Step 3: Conditioning on the first-sale region r_c. For a producing pair (r_p, s_p) and a first-sale region r_c, split the observed output of (r_p, s_p) into the part that is *first sold* to total demand in r_c. Define the corresponding first-sale activity level as the sum of: (i) intermediate sales from (r_p, s_p) to sectors located in r_c, and (ii) direct final demand deliveries from (r_p, s_p) to final demand in r_c:

$$x_{(r_p,s_p),r_c}(t) = \sum_{s_c \in \mathrm{Sec}} Z_{(r_p,s_p),(r_c,s_c)}(t) + \sum_{y \in FD} Y_{(r_p,s_p),(r_c,y)}(t). \tag{22}$$

Direct final demand part of the first-sale slice. Within this first-sale slice, define the share that is delivered *directly* to final demand in r_c:

$$\theta_{(r_p,s_p),r_c}(t) = \frac{\sum_{y \in FD} Y_{(r_p,s_p),(r_c,y)}(t)}{x_{(r_p,s_p),r_c}(t)}. \tag{23}$$

Intermediate-sales part of the first-sale slice. The remaining share $1 - \theta_{(r_p,s_p),r_c}(t)$ is first sold to sectors located in r_c. Define the corresponding first-sale shares across downstream sectors in r_c:

$$\phi_{(r_p,s_p),(r_c,s_c)}(t) = \frac{Z_{(r_p,s_p),(r_c,s_c)}(t)}{x_{(r_p,s_p),r_c}(t)}, \qquad \sum_{s_c \in \mathrm{Sec}} \phi_{(r_p,s_p),(r_c,s_c)}(t) = 1 - \theta_{(r_p,s_p),r_c}(t), \quad \text{for } x_{(r_p,s_p),r_c}(t) > 0. \tag{24}$$

Zero-output convention. If $x_{(r_p,s_p),r_c}(t) = 0$, we set $\theta_{(r_p,s_p),r_c}(t) = 0$ and $\phi_{(r_p,s_p),(r_c,s_c)}(t) = 0$ for all $s_c \in \text{Sec}$, so that all conditioned UT(FDa) shares for the slice (r_p, s_p, r_c) evaluate to zero.

Destination distribution of the first-sale slice. The first-sale slice $x_{(r_p,s_p),r_c}(t)$ is composed of two *mutually exclusive* parts: (i) a **direct-to-final demand** part observed in $Y_{(r_p,s_p),(r_c,y)}(t)$, and (ii) an **intermediate-sales** part observed in $Z_{(r_p,s_p),(r_c,s_c)}(t)$ and subsequently propagated downstream.

Fix an ultimate consuming region r_f. The share of the first-sale slice that ultimately ends in final demand in r_f is then the sum of:

- a **direct** contribution, which can only end in $r_f = r_c$; and
- a **downstream-propagated** contribution, obtained by distributing intermediate sales across downstream sectors in r_c and inheriting their ultimate destination shares $\pi_{(r_c,s_c),r_f}(t)$ from Step 2.

Accordingly, define for each $r_f \in \text{Reg}$:

$$\kappa_{(r_c,r_p,s_p),\,r_f}(t) = \underbrace{\delta_{r_f r_c}\ \theta_{(r_p,s_p),r_c}(t)}_{\text{direct delivery to final demand in } r_c} + \underbrace{\sum_{s_c \in \text{Sec}} \phi_{(r_p,s_p),(r_c,s_c)}(t)\ \pi_{(r_c,s_c),r_f}(t)}_{\text{downstream propagation via sectors located in } r_c} \quad . \tag{25}$$

Where $\delta_{r_f r_c}$:

$$\delta_{r_f r_c} = \begin{cases} 1, & \text{if } r_f = r_c, \\ 0, & \text{if } r_f \neq r_c. \end{cases}$$

It ensures that the direct-to-final demand share $\theta_{(r_p,s_p),r_c}(t)$ contributes only to the ultimate destination $r_f = r_c$ (i.e., direct final demand deliveries observed in $Y_{(r_p,s_p),(r_c,y)}(t)$ remain in consuming region r_c). Here $\kappa(t)$ is indexed with row key (r_p, s_p, r_c) and column key r_f, i.e., it distributes the first-sale slice in r_c over ultimate consuming regions r_f.

Normalization. Summing Eq. (25) over $r_f \in \text{Reg}$ yields

$$\sum_{r_f \in \text{Reg}} \kappa_{(r_c,r_p,s_p),\,r_f}(t) = \theta_{(r_p,s_p),r_c}(t) + \sum_{s_c \in \text{Sec}} \phi_{(r_p,s_p),(r_c,s_c)}(t) = 1, \tag{26}$$

where we used $\sum_{r_f \in \text{Reg}} \pi_{(r_c,s_c),r_f}(t) = 1$ (Step 2) and $\sum_{s_c \in \text{Sec}} \phi_{(r_p,s_p),(r_c,s_c)}(t) = 1 - \theta_{(r_p,s_p),r_c}(t)$ from Eq. (24). Hence $\kappa_{(r_c,r_p,s_p),\,r_f}(t)$ is a distribution of the first-sale slice over ultimate consuming regions r_f.

4.2.2.4 Step 4: UT(FDa) shares for each L_2 functional unit. Let $aSoCC^{L_{1.a}}_{r_f}(t)$ be any admissible L_1 weighting as reported in Table 5 [Paulillo et al., 2026, Yang and Paulillo, 2025]:

$$\sum_{r_f \in \text{Reg}} aSoCC^{L_{1.a}}_{r_f}(t) = 1. \tag{27}$$

Define the total final demand in consuming region r_f:

$$\sum_{r' \in \text{Reg}} \sum_{s' \in \text{Sec}} \sum_{y \in FD} Y_{(r',s'),(r_f,y)}(t). \tag{28}$$

$L_{2.a.b}$**: total production by** (r_p, s_p)**.** For fixed (r_p, s_p), define the activity level

$$x_{(r_p,s_p)}(t) = \sum_{r_c \in \text{Reg}} x_{(r_p,s_p),r_c}(t),$$

and the corresponding ultimate destination distribution as the first-sale weighted aggregation:

$$\kappa_{(r_p,s_p),\,r_f}(t) = \frac{\sum\limits_{r_c \in \text{Reg}} x_{(r_p,s_p),r_c}(t)\ \kappa_{(r_c,r_p,s_p),\,r_f}(t)}{x_{(r_p,s_p)}(t)}. \tag{29}$$

- **Two-step option (with explicit L_1 weighting).**

$$UT(FDa)^{L_{2.a.b}}_{(r_p,s_p)}(t) = \sum_{r_f \in \text{Reg}} aSoCC^{L_{1.a}}_{r_f}(t)\ \frac{x_{(r_p,s_p)}(t)\ \kappa_{(r_p,s_p),\,r_f}(t)}{\sum\limits_{r' \in \text{Reg}} \sum\limits_{s' \in \text{Sec}} \sum\limits_{y \in \text{FD}} Y_{(r',s'),(r_f,y)}(t)}. \tag{30}$$

- **One-step option (no explicit L_1 weighting).** Alternatively, one may normalize directly to *global* final demand by replacing the region-specific denominator $\sum_{r',s',y} Y_{(r',s'),(r_f,y)}(t)$ by the global total $\sum_{r_f,r',s',y} Y_{(r',s'),(r_f,y)}(t)$. Since from Eq (26), $\sum_{r_f \in \text{Reg}} \kappa_{(r_p,s_p),\,r_f}(t) = 1$, the activity level attributed across all consuming regions is

$$\sum_{r_f \in \text{Reg}} x_{(r_p,s_p)}(t)\, \kappa_{(r_p,s_p),\,r_f}(t) = x_{(r_p,s_p)}(t),$$

  so the corresponding one-step UT(FDa) share is

$$UT(FDa)^{L_{2.a.b}}_{(r_p,s_p)}(t) = \frac{x_{(r_p,s_p)}(t)}{\sum\limits_{r_f \in \text{Reg}} \sum\limits_{r' \in \text{Reg}} \sum\limits_{s' \in \text{Sec}} \sum\limits_{y \in \text{FD}} Y_{(r',s'),(r_f,y)}(t)}. \tag{31}$$

$L_{2.b.b}$**: total production by** (r_p, s_p) **directly sold to total demand in** r_c**.** For fixed (r_p, s_p, r_c), use directly the first-sale activity level $x_{(r_p,s_p),r_c}(t)$ from Eq. (22) and the corresponding ultimate destination shares $\kappa_{(r_c,r_p,s_p),\,r_f}(t)$ from Eq. (25).

- **Two-step option (with explicit L_1 weighting).**

$$UT(FDa)^{L_{2.b.b}}_{(r_p,s_p),r_c}(t) = \sum_{r_f \in \text{Reg}} aSoCC^{L_{1.a}}_{r_f}(t)\ \frac{x_{(r_p,s_p),r_c}(t)\ \kappa_{(r_c,r_p,s_p),\,r_f}(t)}{\sum\limits_{r' \in \text{Reg}} \sum\limits_{s' \in \text{Sec}} \sum\limits_{y \in \text{FD}} Y_{(r',s'),(r_f,y)}(t)}. \tag{32}$$

- **One-step option (no explicit L_1 weighting).** Alternatively, one may normalize directly to *global* final demand. Since from Eq (26), $\sum_{r_f \in \text{Reg}} \kappa_{(r_c,r_p,s_p),\,r_f}(t) = 1$, the activity level attributed across all consuming regions is

$$\sum_{r_f \in \text{Reg}} x_{(r_p,s_p),r_c}(t)\, \kappa_{(r_c,r_p,s_p),\,r_f}(t) = x_{(r_p,s_p),r_c}(t),$$

  so the corresponding one-step UT(FDa) share is

$$UT(FDa)^{L_{2.b.b}}_{(r_p,s_p),r_c}(t) = \frac{x_{(r_p,s_p),r_c}(t)}{\sum\limits_{r_f \in \text{Reg}} \sum\limits_{r' \in \text{Reg}} \sum\limits_{s' \in \text{Sec}} \sum\limits_{y \in \text{FD}} Y_{(r',s'),(r_f,y)}(t)}. \tag{33}$$

$L_{2.c.b}$: total demand in r_c of outputs of sector s_p (wherever produced). For fixed (s_p, r_c), define the corresponding activity level as the total first-sale slice into r_c from all producers of sector s_p:

$$\sum_{r_p \in \mathrm{Reg}} x_{(r_p,s_p),r_c}(t).$$

Define the corresponding ultimate destination distribution as the producing-region weighted aggregation:

$$\kappa_{(r_c,s_p),\,r_f}(t) = \frac{\sum\limits_{r_p \in \mathrm{Reg}} x_{(r_p,s_p),r_c}(t)\ \kappa_{(r_c,r_p,s_p),\,r_f}(t)}{\sum\limits_{r_p \in \mathrm{Reg}} x_{(r_p,s_p),r_c}(t)}. \tag{34}$$

- **Two-step option (with explicit L_1 weighting).**

$$UT(FDa)^{L_{2.c.b}}_{(r_c,s_p)}(t) = \sum_{r_f \in \mathrm{Reg}} aSoCC^{L_{1.a}}_{r_f}(t)\ \frac{\sum\limits_{r_p \in \mathrm{Reg}} x_{(r_p,s_p),r_c}(t)\ \kappa_{(r_c,s_p),\,r_f}(t)}{\sum\limits_{r' \in \mathrm{Reg}} \sum\limits_{s' \in \mathrm{Sec}} \sum\limits_{y \in \mathrm{FD}} Y_{(r',s'),(r_f,y)}(t)}. \tag{35}$$

- **One-step option (no explicit L_1 weighting).** Alternatively, one may normalize directly to *global* final demand. Since from Eq (26), $\sum_{r_f \in \mathrm{Reg}} \kappa_{(r_c,s_p),\,r_f}(t) = 1$, the activity level attributed across all consuming regions is

$$\sum_{r_p \in \mathrm{Reg}} x_{(r_p,s_p),r_c} \sum_{r_f \in \mathrm{Reg}} (t)\kappa_{(r_c,s_p),\,r_f}(t) = \sum_{r_p \in \mathrm{Reg}} x_{(r_p,s_p),r_c}(t),$$

  so the corresponding one-step UT(FDa) share is

$$UT(FDa)^{L_{2.c.b}}_{(r_c,s_p)}(t) = \frac{\sum\limits_{r_p \in \mathrm{Reg}} x_{(r_p,s_p),r_c}(t)}{\sum\limits_{r_f \in \mathrm{Reg}} \sum\limits_{r' \in \mathrm{Reg}} \sum\limits_{s' \in \mathrm{Sec}} \sum\limits_{y \in \mathrm{FD}} Y_{(r',s'),(r_f,y)}(t)}. \tag{36}$$

4.3 UT(GVAa): downstream attribution of upstream value added responsibility via the Leontief inverse

Motivation and principle. Utilitarian allocation methods can be formulated from a *production-anchored* perspective by attaching utility to *value added creation* rather than to final demand. Gross value added (GVA) measures the direct economic value created by each producing sector-region pair and can therefore serve as a production-side proxy for utility [Algunaibet et al., 2019, Balanza et al., 2025]. However, when utility is measured only by unadjusted direct GVA of the studied producing pair (r_p, s_p), then the resulting allocated space corresponds only to that pair's *production-based, scope 1* activity (PBA system boundaries). Such a specification ignores that producing (r_p, s_p)'s output mobilizes upstream suppliers whose *own* scope 1 activities are required to enable that output. It therefore under-allocates space to (r_p, s_p) whenever the intended interpretation is a scope 1–2–3 budget that internalizes upstream supply-chain requirements [Balanza et al., 2025].

To address this, we propose the UT(GVAa) to allocate to a producing activity (r_p, s_p): (i) its own scope 1 aSoCC (based on the value added it directly generates), plus (ii) inherited portions of the scope 1 aSoCCs of all upstream suppliers, proportional to how much of each upstream supplier's value added is mobilized (directly and indirectly) to produce (r_p, s_p)'s output (i.e., aSoCC for (r_p, s_p) Scope 2 and upstream Scope 3 environmental impacts).

This value-added-based propagation has been previously described in the context of supply-chain responsibilities [Lenzen et al., 2007, Gopalakrishnan et al., 2021, Gopalakrishnan, 2022]. That literature further discusses how responsibility for supply-chain-related impacts may translate into obligations to finance or implement mitigation across tiers. In contrast, AESA and *UNCASExt* stop at the prior step, quantifying how much carrying capacity is allocated to an actor for its full life-cycle impacts, without prescribing how mitigation burdens should be distributed across actors whose activities jointly determine whether each sector aSoCC is respected.

Functional units. The UT(GVAa) allocation method is compatible with the three *UNCASExt* L_2 functional units (Table 2):

- $L_{2.a.b}$: Total production of goods and services by sector s_p in producing region(s) r_p in year t.
- $L_{2.b.b}$: Total production of goods and services by sector s_p in producing region(s) r_p directly supplied to total demand in region(s) r_c in year t.
- $L_{2.c.b}$: Total demand in region(s) r_c in year t of goods and services produced by sector s_p.

Notation and indices. A producing sector–region pair is denoted (r, s). We distinguish (r_p, s_p) for the downstream producing pair whose UT(GVAa) is reported, (r_u, s_u) for an upstream supplier, and r_c for the first-sale region (where the studied outputs are first sold to total demand: intermediate and/or final demand).

Let $Z(t)$ be the transaction matrix, $Y(t)$ the final demand matrix, $x(t)$ the gross output vector, and u the vector of ones. Let $F_v(t)$ be the factor production value added block representing the total amount of value added category v. Define the gross value added (GVA) created by (r, s) as the sum of its value-added components:

$$\sum_{v \in VA} F_{v,(r,s)}(t).$$

Following `pymrio` notations [Stadler, 2021], define the input-coefficient matrix and the Leontief inverse by

$$A(t) = Z(t)\,\widehat{x}(t)^{-1}, \qquad L(t) = \big(I - A(t)\big)^{-1}. \tag{37}$$

4.3.1 Step 1: Upstream value-added origin shares embodied in one unit of downstream production.

Step 1 constructs, for each downstream producing pair (r_p, s_p), a distribution over upstream suppliers (r_u, s_u) describing how the value added embodied in *one unit* of output of (r_p, s_p) is distributed across upstream suppliers, accounting for all upstream tiers.

Step 1.a: Direct value-added shares of output. Define the direct value-added share of output of (r, s) by

$$\alpha_{(r,s)}(t) = \frac{\sum\limits_{v \in VA} F_{v,(r,s)}(t)}{x_{(r,s)}(t)}. \tag{38}$$

Collect these coefficients into the diagonal matrix $\widehat{\alpha}(t)$.

Zero-output convention. If $x_{(r,s)}(t) = 0$, we set $\alpha_{(r,s)}(t) = 0$.

Step 1.b: Upstream origin matrix. Define the matrix of upstream value-added origin shares by

$$\mathit{\Omega}(t) = \widehat{\alpha}(t)\, L(t), \tag{39}$$

and denote its elements by $\omega_{(r_u,s_u),(r_p,s_p)}(t)$. Equivalently, elementwise,

$$\omega_{(r_u,s_u),(r_p,s_p)}(t) = \alpha_{(r_u,s_u)}(t)\; L_{(r_u,s_u),(r_p,s_p)}(t). \tag{40}$$

$\omega_{(r_u,s_u),(r_p,s_p)}(t)$ is interpretable as: *the share of the value added embodied (directly and indirectly, through all upstream producers) in one unit of output of* (r_p, s_p) *that is created by upstream supplier* (r_u, s_u)*.*

Step 1.c: Normalization. For each fixed downstream pair (r_p, s_p), the upstream origin shares form a distribution over upstream suppliers:

$$\sum_{r_u \in \text{Reg}} \sum_{s_u \in \text{Sec}} \omega_{(r_u,s_u),(r_p,s_p)}(t) = 1. \tag{41}$$

i.e., as the value-added share vector satisfies

$$\alpha(t)^{\top} = u^{\top}(I - A(t)),$$

so

$$\alpha(t)^{\top} L(t) = u^{\top}(I - A(t))(I - A(t))^{-1} = u^{\top}.$$

Taking the (r_p, s_p) component yields

$$\sum_{r_u \in \text{Reg}} \sum_{s_u \in \text{Sec}} \alpha_{(r_u,s_u)}(t)\; L_{(r_u,s_u),(r_p,s_p)}(t) = 1.$$

4.3.2 Step 2: Conditioning on the first-sale region r_c.

For functional units that condition the studied activity on a first-sale region r_c, define the corresponding restricted activity level:

$$x_{(r_p,s_p),r_c}(t) = \sum_{s_c \in \text{Sec}} Z_{(r_p,s_p),(r_c,s_c)}(t) + \sum_{y \in FD} Y_{(r_p,s_p),(r_c,y)}(t). \tag{42}$$

Since $\omega_{(r_u,s_u),(r_p,s_p)}(t)$ is defined per unit of downstream production, restricting the studied activity from $x_{(r_p,s_p)}(t)$ to $x_{(r_p,s_p),r_c}(t)$ scales embodied upstream value added linearly: the amount of upstream value added attributed to the restricted activity is $x_{(r_p,s_p),r_c}(t)\, \omega_{(r_u,s_u),(r_p,s_p)}(t)$.

4.3.3 Step 3: UT(GVAa) shares for each L_2 functional unit.

Let the total value added created in producing region r_u be

$$\sum_{s_u \in \text{Sec}} \sum_{v \in VA} F_{v,(r_u,s_u)}(t).$$

$L_{2.a.b}$**: total production by** (r_p, s_p)**.**

- **Two-step option (with explicit** L_1 **weighting).**

$$UT(GVAa)^{L_{2.a.b}}_{(r_p,s_p)}(t) = \sum_{r_u \in \text{Reg}} aSoCC^{L_{1.b}}_{r_u}(t)\; \frac{x_{(r_p,s_p)}(t) \sum_{s_u \in \text{Sec}} \omega_{(r_u,s_u),(r_p,s_p)}(t)}{\sum_{s_u \in \text{Sec}} \sum_{v \in VA} F_{v,(r_u,s_u)}(t)}. \tag{43}$$

- **One-step option (no explicit L_1 weighting).** Alternatively, one may normalize directly to *global* value added by replacing the region-specific denominator $\sum_{s_u,v} F_{v,(r_u,s_u)}(t)$ by the global total $\sum_{r_u,s_u,v} F_{v,(r_u,s_u)}(t)$. Since from Eq (41), $\sum_{r_u,s_u} \omega_{(r_u,s_u),(r_p,s_p)}(t) = 1$, the activity level attributed across all upstream suppliers is

$$\sum_{r_u,s_u} x_{(r_p,s_p)}(t)\, \omega_{(r_u,s_u),(r_p,s_p)}(t) = x_{(r_p,s_p)}(t),$$

so the corresponding one-step UT(GVAa) share is

$$UT(GVAa)^{L_{2.a.b}}_{(r_p,s_p)}(t) = \frac{x_{(r_p,s_p)}(t)}{\sum\limits_{r_u\in\text{Reg}} \sum\limits_{s_u\in\text{Sec}} \sum\limits_{v\in VA} F_{v,(r_u,s_u)}(t)} = \frac{x_{(r_p,s_p)}(t)}{\sum\limits_{r_f\in\text{Reg}} \sum\limits_{r'\in\text{Reg}} \sum\limits_{s'\in\text{Sec}} \sum\limits_{y\in\text{FD}} Y_{(r',s'),(r_f,y)}(t)}. \tag{44}$$

See Section 4.4 for the equivalence between one-step UT(FDa) and one-step UT(GVAa).

$L_{2.b.b}$: **total production by (r_p, s_p) directly sold to total demand in r_c.**

- **Two-step option (with explicit L_1 weighting).**

$$UT(GVAa)^{L_{2.b.b}}_{(r_p,s_p),r_c}(t) = \sum_{r_u\in\text{Reg}} aSoCC^{L_{1.b}}_{r_u}(t)\, \frac{x_{(r_p,s_p),r_c}(t) \sum\limits_{s_u\in\text{Sec}} \omega_{(r_u,s_u),(r_p,s_p)}(t)}{\sum\limits_{s_u\in\text{Sec}} \sum\limits_{v\in VA} F_{v,(r_u,s_u)}(t)}. \tag{45}$$

- **One-step option (no explicit L_1 weighting).** Alternatively, one may normalize directly to *global* value added by replacing the region-specific denominator $\sum_{s_u,v} F_{v,(r_u,s_u)}(t)$ by the global total $\sum_{r_u,s_u,v} F_{v,(r_u,s_u)}(t)$. Since from Eq (41), $\sum_{r_u,s_u} \omega_{(r_u,s_u),(r_p,s_p)}(t) = 1$, the activity level attributed across all upstream suppliers is

$$\sum_{r_u,s_u} x_{(r_p,s_p),r_c}(t)\, \omega_{(r_u,s_u),(r_p,s_p)}(t) = x_{(r_p,s_p),r_c}(t),$$

so the corresponding one-step UT(GVAa) share is

$$UT(GVAa)^{L_{2.b.b}}_{(r_p,s_p),r_c}(t) = \frac{x_{(r_p,s_p),r_c}(t)}{\sum\limits_{r_u\in\text{Reg}} \sum\limits_{s_u\in\text{Sec}} \sum\limits_{v\in VA} F_{v,(r_u,s_u)}(t)} = \frac{x_{(r_p,s_p),r_c}(t)}{\sum\limits_{r_f\in\text{Reg}} \sum\limits_{r'\in\text{Reg}} \sum\limits_{s'\in\text{Sec}} \sum\limits_{y\in\text{FD}} Y_{(r',s'),(r_f,y)}(t)}. \tag{46}$$

See Section 4.4 for the equivalence between one-step UT(FDa) and one-step UT(GVAa).

$L_{2.c.b}$: **total demand in r_c of outputs of sector s_p (wherever produced).**

- **Two-step option (with explicit L_1 weighting).**

$$UT(GVAa)^{L_{2.c.b}}_{(r_c,s_p)}(t) = \sum_{r_u\in\text{Reg}} aSoCC^{L_{1.b}}_{r_u}(t)\, \frac{\sum_{r_p\in\text{Reg}} x_{(r_p,s_p),r_c}(t)\ \sum_{s_u\in\text{Sec}} \omega_{(r_u,s_u),(r_p,s_p)}(t)}{\sum\limits_{s_u\in\text{Sec}} \sum\limits_{v\in VA} F_{v,(r_u,s_u)}(t)}. \tag{47}$$

- **One-step option (no explicit L_1 weighting).** Alternatively, one may normalize directly to *global* value added by replacing the region-specific denominator $\sum_{s_u,v} F_{v,(r_u,s_u)}(t)$ by the global total $\sum_{r_u,s_u,v} F_{v,(r_u,s_u)}(t)$. Since from Eq (41), $\sum_{r_u,s_u} \omega_{(r_u,s_u),(r_p,s_p)}(t) = 1$, the activity level attributed across all upstream suppliers and producing regions

$$\sum_{r_p,r_u,s_u} x_{(r_p,s_p),r_c}(t)\, \omega_{(r_u,s_u),(r_p,s_p)}(t) = \sum_{r_p} x_{(r_p,s_p),r_c}(t),$$

so the corresponding one-step UT(GVAa) share is

$$UT(GVAa)^{L_{2.c.b}}_{(r_c,s_p)}(t) = \frac{\sum\limits_{r_p \in \text{Reg}} x_{(r_p,s_p),r_c}(t)}{\sum\limits_{r_u \in \text{Reg}} \sum\limits_{s_u \in \text{Sec}} \sum\limits_{v \in VA} F_{v,(r_u,s_u)}(t)} = \frac{\sum\limits_{r_p \in \text{Reg}} x_{(r_p,s_p),r_c}(t)}{\sum\limits_{r_f \in \text{Reg}} \sum\limits_{r' \in \text{Reg}} \sum\limits_{s' \in \text{Sec}} \sum\limits_{y \in \text{FD}} Y_{(r',s'),(r_f,y)}(t)}. \tag{48}$$

See Section 4.4 for the equivalence between one-step UT(FDa) and one-step UT(GVAa).

4.4 Equivalence of one-step UT(FDa) and one-step UT(GVAa): introduction of one-step UT(TD)

This subsection shows that global value added equals global final demand. As a consequence, the *one-step* (global-normalization) versions of UT(FDa) and UT(GVAa) are equal for each admissible L_2 functional unit, because they share the *same numerator* and their *denominators are equal*.

4.4.1 Step 1: Global value added equals global final demand.

Let u be the vector of ones of appropriate dimension. The MRIO output identity is

$$x(t) = Z(t)\,u + Y(t)\,u. \tag{49}$$

Premultiplying by $u^\top$ yields total gross output:

$$u^\top x(t) = u^\top Z(t)\,u + u^\top Y(t)\,u. \tag{50}$$

On the input side, total gross output equals intermediate inputs plus total value added:

$$u^\top x(t) = u^\top Z(t)\,u + \sum_{v \in VA} u^\top F_v(t). \tag{51}$$

Subtracting $u^\top Z(t)\,u$ from both (50) and (51) gives

$$\sum_{v \in VA} u^\top F_v(t) = u^\top Y(t)\,u, \tag{52}$$

i.e.,

$$\sum_{r \in \text{Reg}} \sum_{s \in \text{Sec}} \sum_{v \in VA} F_{v,(r,s)}(t) = \sum_{r_f \in \text{Reg}} \sum_{r' \in \text{Reg}} \sum_{s' \in \text{Sec}} \sum_{y \in FD} Y_{(r',s'),(r_f,y)}(t). \tag{53}$$

4.4.2 Step 2: Equality of one-step UT(FDa) and one-step UT(GVAa).

For each admissible L_2 functional unit, both one-step UT(FDa) and one-step UT(GVAa) reduce to the same activity level in the numerator:

- $L_{2.a.b}$: $x_{(r_p,s_p)}(t)$,
- $L_{2.b.b}$: $x_{(r_p,s_p),r_c}(t)$,
- $L_{2.c.b}$: $\sum_{r_p \in \text{Reg}} x_{(r_p,s_p),r_c}(t)$.

As illustrated in the previous subsections, it is the case as the total value added embodied in one unit of output equals that unit's value.

The one-step option of UT(FDa) normalizes by the global sum of final demand,

$$\sum_{r_f \in \text{Reg}} \sum_{r' \in \text{Reg}} \sum_{s' \in \text{Sec}} \sum_{y \in FD} Y_{(r',s'),(r_f,y)}(t),$$

whereas the one-step option of UT(GVAa) normalizes by the global sum of value added,

$$\sum_{r \in \text{Reg}} \sum_{s \in \text{Sec}} \sum_{v \in VA} F_{v,(r,s)}(t).$$

By Eq. (53), these denominators are equal. Since the numerators coincide for each functional unit, the one-step allocations are equal:

$$UT(FDa)^{L_2}_{(\cdot)}(t) = UT(GVAa)^{L_2}_{(\cdot)}(t), \tag{54}$$

for each of the three admissible functional units $L_{2.a.b}$, $L_{2.b.b}$, and $L_{2.c.b}$, with the corresponding index set $(\cdot)$.

Therefore, one-step UT(FDa) and UT(GVAa) are renamed **UT(TD)** to explicitly indicate that it is a **single one-step allocation method focusing on economic utility that is neither consumption- nor production-anchored**.

5 Projecting aSoCCs for prospective AESA studies

This section describes, for L_1 (country-level) and L_2 (sector-level), how *UNCASExt* can, and cannot, currently support prospective AESA studies by projecting aSoCCs.

5.1 Projecting L_1 aSoCCs

At L_1, *UNCASExt* can project all country-level allocation methods: EG(Pop), PR(GDPcap), AR(E_e), AR($E_{e,\text{cap}}$), and PR–HR($E_{e,\text{cap}}$).

Population and GDP enacting metrics rely on World Bank data through 2024 [WorldBank, 2025] and prospective SSP trajectories from the IIASA SSP Scenario Explorer from 2025 onward [International Institute for Applied Systems Analysis, nd]. To ensure internal coherence in prospective AESA, the SSP selected for the prospective life cycle inventories in the ASR numerator (Phase A), for instance via `premise` [Sacchi et al., 2022], should also used for aSoCC computation in the ASR denominator (Phase B) within the same Monte Carlo iteration. Indeed, when population and GDP are used to allocate shares in prospective years, the resulting country-level shares must be consistent with the population and GDP assumptions underlying the prospective world representation used in Phase A.

It should be noted that, for LCIA-based L_1 allocation methods, i.e., AR(E_e), AR($E_{e,\text{cap}}$), and PR–HR($E_{e,\text{cap}}$), environmental enacting metrics are not projected beyond the last available MRIO year.

For AR methods, this is expected because the allocation is intentionally anchored in a fixed historical reference year. AR(E_e) remains fully constant over time, whereas AR($E_{e,\text{cap}}$) fixes each region's acquired rights basis as its per-capita environmental pressure in reference year t^{ar}, while the final allocated shares vary over time because this fixed per-capita basis is applied to the population distribution in year t.

For PR–HR($E_{e,\text{cap}}$), only the population-weighting component can continue to follow SSP trajectories, while the historical responsibility priority factor remains fixed after the last available MRIO year because the cumulative per-capita environmental pressure, computed over the impact category specific responsibility period, cannot be extended beyond the last available year. This represents a limitation, since the historical responsibility priority factor should ideally be updated using projected cumulative per-capita environmental pressure; however, such projections are not currently available.

When L_1 weights are used in two-step allocation paths to target L_2 functional units, the projected L_1 shares can be combined with any L_2 allocation method, irrespective of whether the corresponding L_2 weights are projected via regression or reused from retrospective MRIO years, as described in Section 5.2.

5.2 Projecting L_2 aSoCCs

At L_2, prospective allocation is more constrained because most sector-level enacting metrics rely on MRIO databases.

MRIO-based economic enacting metrics used by utilitarian (UT) methods are limited by MRIO data availability: EXIOBASE 3.10.2 covers 1995–2024 (2023 and 2024 are nowcasted) [Stadler et al., 2018, Stadler et al., 2026], while OECD-ICIO v2025 covers 1995–2022 [OECD, 2023, Yamano et al., 2023]. Therefore, MRIO-based economic enacting metrics must either be projected or reused for prospective years.

MRIO-based environmental enacting metrics used by AR methods do not need to be projected, since they follow the same reference year logic described for L_1: the aSoCC is computed for a given reference year and then remains constant over time.

5.2.1 Prospective MRIO constraint

To project economic enacting metrics, an ideal solution would be to project complete MRIO tables and then compute L_2 enacting metrics under alternative prospective scenarios. Several studies have proposed prospective MRIOs [Beaufils and Wenz, 2022, Cap et al., 2025, Wiebe et al., 2018]. However, this approach raises two main limitations for the present work.

First, following [Wiebe et al., 2018] projecting complete MRIO tables requires detailed supply and use tables, which are not publicly available for EXIOBASE 3.10.2 or OECD-ICIO v2025, for which only input-output tables are accessible. Second, a projected MRIO system used for allocation in the ASR denominator (Phase B) should ideally be aligned with the projected world state underlying the prospective life cycle inventories (LCIs) used in the ASR numerator (Phase A), for example when these inventories are generated with `premise` [Sacchi et al., 2022]. Without such alignment, environmental burdens (Phase A) and allocated carrying capacities (Phase B) may rely on inconsistent representations of prospective technologies and worldwide value chains.

Additionally, for prospective assessments, changes modeled in the foreground scenarios of the studied activity, such as demand levels or production processes, may affect economy-wide trade and production structures and should therefore be reflected in the MRIO world state used to compute allocated shares through MRIO-based enacting metrics. Addressing these feedbacks would require scenario-based MRIO databases that are consistent with the specific prospective foreground scenarios defined for the studied activity in Phase A. This constitutes a dedicated modeling task beyond the scope of this work. Consequently, *UNCASExt* and `pyaesa` don't currently project complete MRIO tables, but instead project or reuse the MRIO-based economic enacting metrics required by the UT allocation equations.

5.2.2 Adopted strategies for MRIO-based economic enacting metrics

Two strategies are retained for prospective MRIO-based economic enacting metrics: regression projection and historical reuse. Regression projection is available for non-adjusted UT economic enacting metrics, namely FD, GVA, and TD. Historical reuse is available for all UT economic enacting metrics and is therefore required for the adjusted allocation methods UT(FDa) and UT(GVAa).

The adjusted allocation methods UT(FDa) and UT(GVAa) cannot be projected through the regression strategy because they depend on the full inter-country and inter-sector MRIO structure. UT(FDa) relies on Ghosh-based downstream propagation of final demand contributions, while UT(GVAa) relies on

Leontief-based upstream propagation of value added responsibilities. Regressing selected totals or shares would not preserve the input-output equilibrium required to compute these propagation mechanisms. Therefore, in `pyaesa`, when regression projection is selected, non-adjusted UT methods are projected through regression whereas UT(FDa) and UT(GVAa) remain represented through historical reuse.

5.2.3 Regression projection of non-adjusted economic enacting metrics

The regression projection has a layered structure. It uses ordinary least squares (OLS) level regressions for country-level economic enacting metrics and OLS log-ratio regressions for within-country shares. This approach is inspired by the MRIO GDP scaling method proposed by [Wiebe et al., 2018]. OLS level regressions follow standard linear regression and time-trend modeling practice [Wooldridge, 2009, Greene, 2003, Seber and Lee, 2003]. The log-ratio regressions follow compositional-data analysis, where positive shares constrained to sum to one are modeled through ratios between components [Aitchison, 1982, Pawlowsky-Glahn et al., 2015].

Let T_{reg} denote the historical regression window. In the `pyaesa` implementation, it is set by default to the historical coverage of the two MRIO sources used, EXIOBASE 3.10.2 and OECD-ICIO v2025: 1995–2022. The nowcasted EXIOBASE years 2023 and 2024 are therefore excluded by default, although any other historical regression window within the available MRIO coverage can be selected.

(i) The first layer projects the country-level economic enacting metric required by the selected method and functional unit. Depending on the route, this can correspond to final demand by final demand region, gross value added by producing region, total output by producing region, or total demand absorption by consuming region. For $r \in Reg$, the country-level economic enacting metric $EM_r(t)$ is modeled as an OLS regression on GDP [Wooldridge, 2009, Greene, 2003, Seber and Lee, 2003]:

$$EM_r(t) = \alpha_r + \beta_r GDP_r(t) + \varepsilon_r(t), \qquad t \in T_{\text{reg}}, \tag{55}$$

where $GDP_r(t)$ is the GDP of region r in year t, α_r and β_r are the OLS intercept and GDP coefficient, and $\varepsilon_r(t)$ is the residual term. The projected country-level economic enacting metric for target year t^* is then computed as:

$$\widehat{EM}_r(t^*) = \max\left(0, \widehat{\alpha}_r + \widehat{\beta}_r GDP_r(t^*)\right). \tag{56}$$

The lower clipping at zero is applied because economic enacting metrics should be nonnegative by construction. Retrospective GDP values are taken from the World Bank through 2024 [WorldBank, 2025], while prospective GDP values are taken from SSP trajectories [International Institute for Applied Systems Analysis, nd].

(ii) The second layer projects sector shares within the projected country-level economic enacting metric. Sector-level economic enacting metrics are not projected independently as levels because independent level regressions would not preserve additivity within each country. Instead, in `pyaesa` implementation, the sector-level economic enacting metrics are first expressed as shares of the corresponding country-level economic enacting metric, then projected using OLS log-ratio regressions in time.

For a studied sector s in country r, let $p_{r,s}(t)$ be the observed share of this sector in the country-level economic enacting metric:

$$p_{r,s}(t) = \frac{EM_{r,s}(t)}{EM_r(t)}. \tag{57}$$

Remaining sectors are represented by a residual category, denoted REST, with share $p_{r,\text{REST}}(t) = 1 - p_{r,s}(t)$. This limits computational needs by avoiding the explicit modeling of all sector-country pairs, while preserving additivity between the studied sector and the residual category. The log-ratio projection is then fitted between the studied sector and the residual category [Aitchison, 1982, Pawlowsky-Glahn et al., 2015]. In the `pyaesa` implementation, the baseline category $s_0(r)$ is selected as the category with the largest number of strictly positive observations in T_{reg}; if both have the same number, the first category in lexicographic order is used. Let $c(r)$ denote the other category, i.e., the non-baseline category.

The log-ratio model is:

$$\log\left(\frac{p_{r,c(r)}(t)}{p_{r,s_0(r)}(t)}\right) = \alpha_{r,c} + \beta_{r,c} t + \varepsilon_{r,c}(t), \tag{58}$$

where $\alpha_{r,c}$ and $\beta_{r,c}$ are the OLS intercept and time-trend coefficient for the fitted log ratio, and $\varepsilon_{r,c}(t)$ is the residual term. For target year t^*, let $\eta_{r,c}(t^*)$ denote the fitted log-ratio predictor for the non-baseline category $c(r)$ relative to the baseline category $s_0(r)$:

$$\eta_{r,c}(t^*) = \widehat{\alpha}_{r,c} + \widehat{\beta}_{r,c} t^*, \qquad \eta_{r,s_0(r)}(t^*) = 0. \tag{59}$$

The projected shares are reconstructed by baseline-referenced normalization:

$$\widehat{p}_{r,c(r)}(t^*) = \frac{\exp\left(\eta_{r,c}(t^*)\right)}{1 + \exp\left(\eta_{r,c}(t^*)\right)}, \qquad \widehat{p}_{r,s_0(r)}(t^*) = \frac{1}{1 + \exp\left(\eta_{r,c}(t^*)\right)}. \tag{60}$$

The projected sector-level economic enacting metric for the studied sector is then:

$$\widehat{EM}_{r,s}(t^*) = \widehat{EM}_r(t^*) \widehat{p}_{r,s}(t^*). \tag{61}$$

(iii) The third layer is optional. It is used only when the functional unit requires producing region filtering in addition to the consuming region and producing sector dimensions already represented by the first two layers. In other words, after projecting the country-level economic enacting metric and the share associated with the studied producing sector, this third layer splits the resulting economic enacting metric according to the producing region of that sector. This applies only to UT(FD) for functional units *L2.a.a* and *L2.b.a*, and to UT(TD) for functional unit *L2.b.b*.

For a studied producing region r_p within the economic enacting metric indexed by consuming region r_c and producing sector s_p, $EM_{r_c,s_p}(t)$, let $q_{r_c,s_p,r_p}(t)$ be the observed producing region share:

$$q_{r_c,s_p,r_p}(t) = \frac{EM_{r_c,s_p,r_p}(t)}{EM_{r_c,s_p}(t)}. \tag{62}$$

The remaining producing regions are represented by a residual category, denoted REST, with share $q_{r_c,s_p,\mathrm{REST}}(t) = 1 - q_{r_c,s_p,r_p}(t)$. The baseline producing region category $r_{p,0}(r_c, s_p)$ is selected using the same rule as for producing sectors: the category with the largest number of strictly positive observations in the regression window is used, and if both have the same number, the first category in lexicographic order is used. Let $d(r_c, s_p)$ denote the non-baseline category.

The producing region log-ratio model is:

$$\log\left(\frac{q_{r_c,s_p,d(r_c,s_p)}(t)}{q_{r_c,s_p,r_{p,0}(r_c,s_p)}(t)}\right) = \alpha^q_{r_c,s_p,d} + \beta^q_{r_c,s_p,d} t + \varepsilon^q_{r_c,s_p,d}(t). \tag{63}$$

where $\alpha^q_{r_c,s_p,d}$ and $\beta^q_{r_c,s_p,d}$ are the OLS intercept and time-trend coefficient for the producing region share log ratio, and $\varepsilon^q_{r_c,s_p,d}(t)$ is the residual term. For target year t^*, let $\eta^q_{r_c,s_p,d}(t^*)$ denote the fitted log-ratio predictor for the non-baseline producing region category $d(r_c, s_p)$ relative to the baseline producing region category $r_{p,0}(r_c, s_p)$:

$$\eta^q_{r_c,s_p,d}(t^*) = \widehat{\alpha}^q_{r_c,s_p,d} + \widehat{\beta}^q_{r_c,s_p,d} t^*, \qquad \eta^q_{r_c,s_p,r_{p,0}(r_c,s_p)}(t^*) = 0. \tag{64}$$

The projected producing region shares are reconstructed by baseline-referenced normalization, following the same log-ratio logic used for the producing sector share projection [Aitchison, 1982, Pawlowsky-Glahn et al., 2015]:

$$\widehat{q}_{r_c,s_p,d(r_c,s_p)}(t^*) = \frac{\exp\left(\eta^q_{r_c,s_p,d}(t^*)\right)}{1+\exp\left(\eta^q_{r_c,s_p,d}(t^*)\right)}, \qquad \widehat{q}_{r_c,s_p,r_{p,0}(r_c,s_p)}(t^*) = \frac{1}{1+\exp\left(\eta^q_{r_c,s_p,d}(t^*)\right)}. \tag{65}$$

The projected economic enacting metric indexed by consuming region, producing sector, and producing region is:

$$\widehat{EM}_{r_c,s_p,r_p}(t^*) = \widehat{EM}_{r_c,s_p}(t^*)\widehat{q}_{r_c,s_p,r_p}(t^*). \tag{66}$$

The resulting projection structure depends on the selected UT allocation method and functional unit. For UT(FD), final demand economic enacting metrics are first projected by final demand region (r_f), followed by the producing sector share within each r_f; for *L2.a.a* and *L2.b.a* only, the optional third layer further filters this projected (r_f, s_p) economic enacting metric by producing region (r_p). For UT(GVA), gross value added economic enacting metrics are first projected by producing region (r_p), followed by the producing sector share within each r_p; no additional producing region layer is required because r_p is already specified at the first layer. For UT(TD), the structure depends on the functional unit: *L2.a.b* projects total output by producing region (r_p) and then producing sector shares within each r_p, whereas *L2.b.b* and *L2.c.b* project total demand by consuming region (r_c) and then producing sector shares within each r_c; only *L2.b.b* additionally requires the optional producing region layer within each (r_c, s_p) pair.

This construction preserves additivity by design: the studied producing sector share and the residual producing sector share sum to one, and, when producing region filtering is required, the studied producing region share and the residual producing region share also sum to one. The projected sector-level economic enacting metric is therefore consistent with the projected country-level economic enacting metric, and the projected producing region economic enacting metric is consistent with the projected producing sector economic enacting metric. The underlying assumptions are that prospective GDP influences the prospective level of the country-level economic enacting metric, while retrospective MRIO composition trends describe the prospective producing sector and producing region shares within that economic enacting metric.

The resulting projection is only partially SSP-dependent. SSP dependence enters through the projected country-level economic enacting metric, because the fitted OLS level regression is evaluated using the SSP GDP trajectory of the corresponding MRIO region. Shares within that economic enacting metric are projected from historical MRIO composition trends as a function of time. Producing sector and producing region economic enacting metrics are nevertheless linked to the SSP scenario because they are obtained by multiplying the SSP GDP-driven country-level economic enacting metric by projected shares. The method is therefore consistent with SSP macroeconomic trajectories where GDP is used, but it does not impose scenario-specific changes in relative producing sector lists, technology, trade structure, or inter-industry relationships. These follow historical trends.

5.2.4 Historical reuse of MRIO-based economic enacting metrics

Historical reuse assigns an observed retrospective MRIO-based economic structure to a prospective target year. It is always used for UT(FDa) and UT(GVAa) (as regression projection is not applied). In `pyaesa` it can also be used optionality for non-adjusted UT economic enacting metrics instead of regression projection. By default, for historical reuse the `pyaesa` implementation uses all available historical years from the selected MRIO source as candidate reuse years (i.e., 1995–2022 for EXIOBASE 3.10.2 and

OECD-ICIO v2025). However it is possible to restrict historical reuse to a selected subset of retrospective year(s).

In the uncertainty assessment phase, each Monte Carlo iteration draws one reuse year with equal probability among the selected set of reuse years. For one-step L_2 allocation paths, historical reuse directly assigns the retrospective L_2 allocated share from the selected reuse year to the prospective target year. For two-step allocation paths, the retrospective within-country L_2 share is reused and then combined with the target year L_1 allocated share, which can still vary with the prospective year and SSP scenario according to Section 5.1. Because the reused within-country L_2 share remains normalized within each country, this combination preserves additivity within the country-level allocation structure. However, ideally, both the L_1 allocated shares and the L_2 MRIO-based economic enacting metrics should be prospective and mutually consistent with the Phase A prospective scenarios.

6 Uncertainty assessment

6.1 Monte Carlo propagation

Monte Carlo simulation is used to propagate uncertainty sources. Since the accuracy of Monte Carlo results increases with the number of iterations, convergence tests are carried out to ensure that uncertainty measures (e.g., mean or median) remain 'sufficiently stable' (arbitrary) when increasing the number of iterations [Rosenbaum et al., 2017]. In the `pyaesa` implementation, convergence is assessed for the mean ASR and the frequency of transgression (i.e., the share of Monte Carlo runs for which $ASR > 1$) for each year and impact category. By default, convergence is considered reached when these indicators remain within a 5% relative tolerance over 10 000 consecutive runs, although both the tolerance and the required number of stable runs can be modified by the user.

Let $k \in \{1, \ldots, N_{\mathrm{mc}}\}$ denote a Monte Carlo iteration and t a studied year. When an uncertainty source affects several years, impact categories, or allocation methods, the same sampled parameter is reused within iteration k to preserve internal consistency.

Phase A and Phase B can be propagated independently and then combined at the ASR level. Phase A provides the distribution of environmental burdens, while Phase B provides the distribution of allocated carrying capacities. In prospective studies, scenario consistency should nevertheless be maintained across both phases. In particular, the SSP used to represent the prospective background system in Phase A should match the SSP used for prospective enacting metrics in Phase B (e.g., population or GDP).

6.2 Phase B uncertainty sources

Phase B computes the ASR denominator by multiplying an allocated share of carrying capacity by a carrying capacity. Uncertainty in Phase B is therefore separated into two objects:

1. uncertainty affecting the allocated shared (aSoCC);
2. uncertainty affecting the carrying capacity (CC).

6.2.1 Uncertainty in aSoCCs

Enacting metric uncertainty. Enacting metric uncertainty concerns the data used to operationalize a sharing principle. It includes two distinct sources: uncertainty in environmental enacting metrics and uncertainty in economic enacting metrics.

Environmental enacting metric uncertainty. Environmental enacting metrics are derived from EXIOBASE. This source of uncertainty is classified as parameter uncertainty in Table 9. The approach adopted in the original UNCASE framework [Puig-Samper et al., 2025] is followed, using coefficients of variation (CoVs) derived from consumption-based carbon accounts reported by [Rodrigues et al., 2018]. It should be noted that these CoVs were estimated for greenhouse gas (GHG) emissions. Their application to other impact categories therefore remains a modeling limitation, as underlined by [Puig-Samper et al., 2025].

For an affected base allocation value $aSoCC_e(t)$, the CoV correction is applied to the enacting metric ratio from which the share is computed. Let c_{num} be the CoV associated with the enacting metric in the numerator and c_{den} the CoV associated with the denominator. For an L_1 allocation, the numerator is the environmental enacting metric of the studied country and the denominator is the corresponding global total. For a one-step L_2 allocation, the numerator is the environmental enacting metric of the studied L_2 sector and the denominator is the corresponding global total. For a two-step $L_1 \to L_2$ allocation path, the correction can be applied separately to the L_1 share and to the within-country L_2 share: the L_1 denominator is the global total, whereas the L_2 denominator is the country-level total.

The lower and upper bounds are:

$$aSoCC_e^{\text{low}}(t) = aSoCC_e(t)\frac{1 - c_{\text{num}}}{1 + c_{\text{den}}}, \qquad aSoCC_e^{\text{high}}(t) = aSoCC_e(t)\frac{1 + c_{\text{num}}}{1 - c_{\text{den}}}. \tag{67}$$

The sampled value is drawn from a continuous uniform distribution over this interval:

$$aSoCC_{e,k}(t) = aSoCC_e^{\text{low}}(t) + \nu_{\text{env},k}\left(aSoCC_e^{\text{high}}(t) - aSoCC_e^{\text{low}}(t)\right), \qquad \nu_{\text{env},k} \sim \mathcal{U}(0,1). \tag{68}$$

Economic enacting metric uncertainty. Economic enacting metric uncertainty is represented through inter-MRIO uncertainty. It reflects the differences arising from the choice of MRIO source used to compute MRIO-based economic enacting metrics. This source is classified as both parameter and model uncertainty in Table 9.

Inter-MRIO uncertainty can be propagated only when the studied sector can be defined with a comparable perimeter in both MRIO sources. In the `pyaesa` implementation, this is possible for sectors for which EXIOBASE 3.10.2 and OECD-ICIO v2025 provide compatible sector definitions. It is therefore not available for all sectors, because EXIOBASE and OECD-ICIO differ in their sector classifications. However, following the approach adopted in the UNCASE framework, the `pyaesa` implementation allows for sector disaggregation when a studied sector is available separately in one MRIO source but only reported within a broader sector group in the other MRIO source.

For a given studied sector and year t, let $aSoCC^{\text{EXIO}}(t)$ and $aSoCC^{\text{OECD}}(t)$ denote the allocated shares of carrying capacity obtained from EXIOBASE and OECD-ICIO, respectively. The sampled value is obtained by interpolation between the two MRIO-based results:

$$\nu_{\text{MRIO},k} \sim \mathcal{U}(0,1), \qquad aSoCC_k(t) = aSoCC^{\text{EXIO}}(t) + \nu_{\text{MRIO},k}\left(aSoCC^{\text{OECD}}(t) - aSoCC^{\text{EXIO}}(t)\right). \tag{69}$$

Intra-MRIO uncertainty associated with algorithms used to balance input-output tables is not available in currently available MRIO databases, hence preventing the inclusion of this uncertainty source. To be exact, intra-MRIO variability is provided in Eora MRIO tables (full) [Lenzen et al., 2013] as standard deviation for each monetary entry. Nevertheless, this MRIO database has a heterogeneous classification and is not maintained anymore.

Reference year uncertainty for AR methods. Reference year uncertainty applies only to acquired rights (AR) methods. These methods rely on a historical reference year t^{ar}, which determines the historically acquired environmental pressure used as the allocation basis. The reference year is therefore a normative choice, and several admissible reference years may be considered. In the `pyaesa` implementation, the default is to consider all available reference years from 1995 (the oldest year available in EXIOBASE) to the first studied year, although a subset of reference years can be selected. One reference year is then sampled in each Monte Carlo iteration with equal probability and applied to all studied years in the run. Allocation methods that do not use an AR reference year are not affected by this uncertainty source.

Projection uncertainty from historical reuse. Projection uncertainty from historical reuse applies only when the historical reuse projection mode is used for prospective L_2 economic enacting metrics. It does not apply when regression projection is used. In the `pyaesa` implementation, the default is to consider all historical reuse years from 1995 to 2022, corresponding to the full historical coverage of the two MRIO sources used (EXIOBASE 3.10.2 and OECD ICIO v2025); the nowcasted EXIOBASE 3.10.2 years 2023 and 2024 are therefore excluded by default, although another subset of reuse years can be selected. One reuse year is then sampled in each Monte Carlo iteration with equal probability and applied to all prospective studied years in the run.

Inter-method uncertainty. There is currently no consensus in the AESA literature on the normative choice of sharing principles at each allocation level [Bjørn et al., 2025, Paulillo et al., 2026, Puig-Samper et al., 2025, Ryberg et al., 2020]. As in the original UNCASE framework [Puig-Samper et al., 2025], equal probabilities of being drawn are therefore assigned to all sharing principles and, conditional on the selected sharing principle, to all associated enacting metrics. This hierarchical equal-weight structure ensures a neutral treatment of sharing principles, irrespective of the number of associated enacting metrics, while still capturing uncertainty at both the sharing principle and enacting metric levels. It thereby avoids structural bias, whereby sharing principles associated with a larger number of enacting metrics would otherwise be over-represented. For example, at L_1 the egalitarian sharing principle is associated with a single enacting metric, whereas the prioritarian sharing principle is associated with two enacting metrics.

Each allocation path from the global level to the studied level represents the preference towards a given alternative [Mendoza Beltran et al., 2016, Puig-Samper et al., 2025]. By construction, no sharing principle, nor any enacting metric within a given sharing principle, is favored, which reflects the current state of the AESA literature [Bjørn et al., 2025, Paulillo et al., 2026, Puig-Samper et al., 2025, Ryberg et al., 2020].

However, the adoption of equal probabilities remains a normative assumption. To make this assumption explicit and to support interpretation, the relative importance of the different allocation paths is visualized using a probability tree, as illustrated in Figure 3. This visualization is inspired by the unweighted tree representation proposed by [Verhaeghe et al., 2024]. The probability tree explicitly illustrates the probability of each allocation path, thereby enhancing uncertainty interpretation. Moreover, it substantially increases flexibility for future AESA applications building on the *UNCASExt* framework, as alternative weighting schemes can be implemented via `pyaesa` to reflect different normative positions regarding sharing principles and enacting metrics.

Figure 3 illustrates the probability tree for an $L2.a.a$ functional unit.

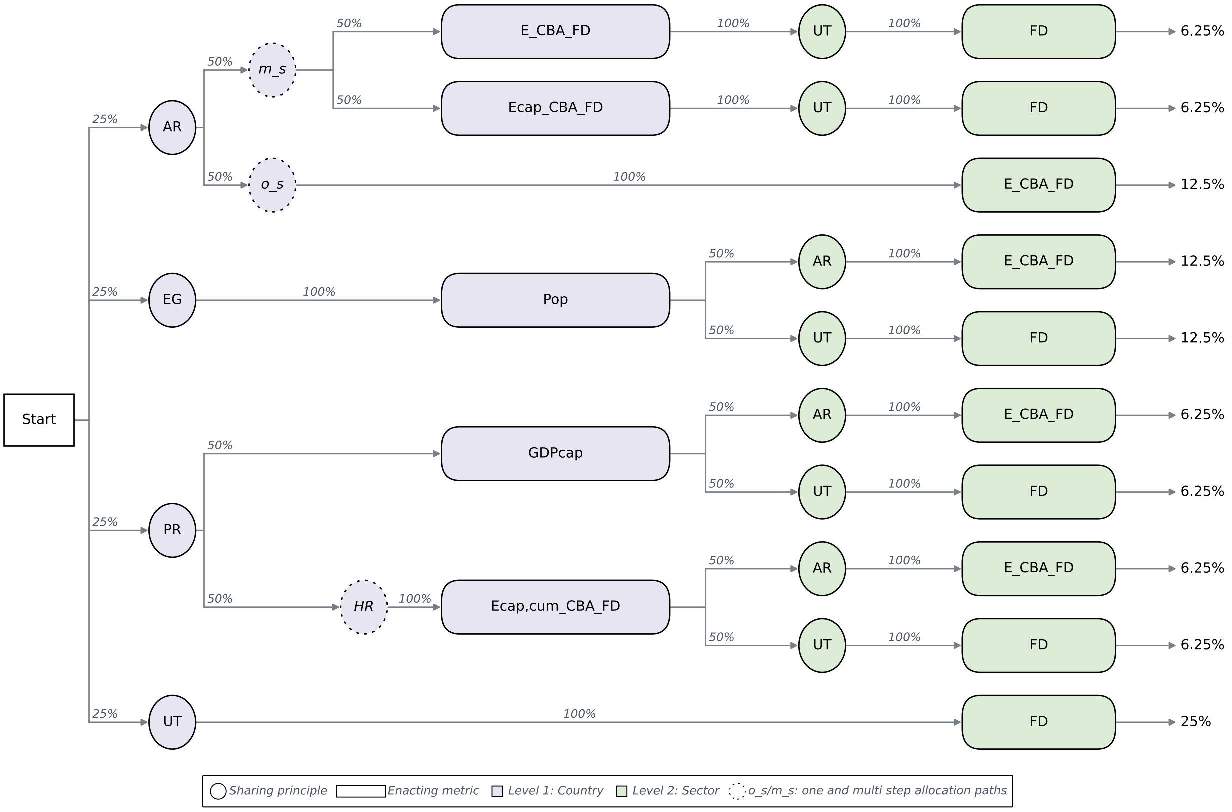


Figure 3: Illustrative equal-weights probability tree for inter-method uncertainty for an *L2.a.a* functional unit. Generated with `pyaesa`.

6.2.2 Uncertainty in carrying capacities

Regarding uncertainty in carrying capacities, two distinct cases are considered: static steady-state carrying capacities and dynamic carrying capacities.

For static steady-state carrying capacities, such as carrying capacities derived from the planetary boundaries framework [Steffen et al., 2015,Richardson et al., 2023] or EF 3.1 [Sala et al., 2020,Sanye Mengual and Sala, 2023], the level or acceptable risk or the uncertainty in carrying capacity threshold level are not sampled within the Monte Carlo simulation. Instead, ASR values are computed and reported for both the minimum and maximum carrying capacity values. This preserves the discernibility of alternative risk levels rather than merging them into a single Monte Carlo distribution. In graphical representations, results based on minimum and maximum carrying capacity values are displayed together to show the range induced by the carrying capacity definition.

For dynamic climate change carrying capacities, scenario uncertainty is propagated through Monte Carlo simulation, since these carrying capacities can be derived from numerous prospective emissions pathways from the Intergovernmental Panel on Climate Change Sixth Assessment Report (IPCC AR6). Details are provided in Appendix C, "Carrying capacities definition".

6.3 Summary of uncertainty sources covered by *UNCASExt*

Table 9 summarizes the main uncertainty and variability sources covered by *UNCASExt* and implemented in `pyaesa`.

Table 9: Sources of uncertainty and variability in absolute environmental sustainability assessment (AESA). Although not exhaustive, the table gathers most of the known uncertainty sources as described in the literature.

AESA phase [Bjørn et al., 2025]	**Uncertainty source**	**Uncertainty type**	**References**	**Covered by UNCASE (original framework)**	**Covered by UNCASExt**
Phase B: Allocate carrying capacities					
Quantification of carrying capacities (CC)	Level of acceptable risk when defining CC threshold	Scenario	[Bjørn et al., 2020, Puig-Samper et al., 2025]	✓	✓
	Choice of CC control variables	Scenario	[Bjørn et al., 2020, Bjørn et al., 2025]	✗	✓ *climate change*
	Inconsistent location of CC control variables on the impact pathway, i.e., Driver-Pressure-State-Impact-Response (DPSIR) framework	Variability between objects	[Bjørn and Hauschild, 2015, Vea et al., 2020]	✗	✓ *climate change*
	Inconsistent time horizon between CC control variables, including steady-state vs. dynamic (cumulative budget) approaches	Temporal variability	[Ryberg et al., 2018, Shukla et al., 2022]	✗	✓ *climate change*
	Uncertainty in CC threshold	Parameter	[Sala et al., 2020, Puig-Samper et al., 2025]	✓	✓
	For the specific case of dynamic (cumulative budget) CCs: differences in yearly CC thresholds under different prospective scenarios (e.g., IAM-SSP-climate pathways scenarios)	(i) Temporal variability (ii) Scenario	[Shukla et al., 2022]	✗	✓ *climate change*
Allocation of shares of carrying capacities (aSoCC)	Choice of system boundaries	Model	[Bjørn et al., 2020, Bjørn et al., 2025]	✗	✓
	Choice of functional unit	Scenario	[Bjørn et al., 2020, Bjørn et al., 2025]	✗	✓
	Normative choice of allocation methods[a]	Scenario	[Puig-Samper et al., 2025, Verhaeghe et al., 2024]	✓	✓
	Inaccurate, non-representative or incomplete enacting metric data	Parameter	[Hauschild et al., 2018, Puig-Samper et al., 2025]	✓[b]	✓[b]
	Differences in yearly enacting metric data under different prospective scenarios (e.g., IAM-SSP-climate pathways scenarios)	(i) Temporal variability (ii) Scenario	[Sacchi et al., 2022, Wiebe et al., 2018]	✗	✓
	Uncertainty in lifetime of substances for the definition of characterization factors applicable to *environmental* enacting metric data	Parameter	[Hauschild et al., 2018, Huijbregts, 1998]	✗	✗
	Differences in yearly characterization factors, applicable to *environmental* enacting metric data, under different prospective scenarios (e.g., climate pathways)	(i) Temporal variability (ii) Scenario	[Clausen et al., 2025, Hauschild et al., 2018]	✗	✗
	Regional differences in characterization factors, applicable to *environmental* enacting metric data	Spatial variability	[Hauschild et al., 2018, Huijbregts, 1998]	✗	✗
Phase C: Interpret results					
Absolute Sustainability Ratio (ASR)	Choice of temporal aggregation of results (year-by-year assessment vs. aggregation over a time horizon)	(i) Temporal variability (ii) Scenario	[Clausen et al., 2025, Lejeune et al., 2026]	✗	✓ *climate change*
	Choice of spatial aggregation of results (region-by-region assessment vs. aggregation across regions – incl. aggregation pre or post application of country-level proritarian methods)	(i) Spatial variability (ii) Scenario	[Paulillo et al., 2026]	✗	✓

IAM = integrated assessment model, CC = carrying capacity, EE-MRIO = environmentally extended multi-regional input-output, LCA = life-cycle assessment, LCI = life-cycle inventory, LCIA = life-cycle impact assessment, SSP = social shared pathway.

Definitions of the seven main types of uncertainty sources [Björklund, 2002, Hauschild et al., 2018, Huijbregts, 1998]: **Variability** reflects inherent heterogeneity in the studied activity and is subdivided into (i) temporal variability (changes over time), (ii) spatial variability (differences across locations), and (iii) variability between objects (differences between technologies, products, or actors, etc.). **Parameter uncertainty** arises from imperfect knowledge of quantitative input values, such as inventory data or characterization factors. **Model uncertainty** results from structural assumptions and simplifications in models, such as the handling of system boundaries and multi-functionality. **Scenario uncertainty** stems from normative choices such as the selected life-cycle impact assessment (LCIA) methods. **Epistemological uncertainty** reflects lack of relevant knowledge. **Mistakes** corresponds to unintended errors such as unit conversion errors. **Relevance uncertainty** concerns, for example, the accuracy or representativeness of impact categories with respect to the area of protection. The three last sources may affect all phases of an AESA study. However, they are not explicitly listed in the table, as they are not commonly subject to systematic probabilistic assessment.

[a] Including the choice of historical reference year for the acquired rights (AR) sharing principle and the responsibility period for the historical responsibility (HR) sharing principle [Paulillo and Sanyé-Mengual, 2024, Paulillo et al., 2026, Ryberg et al., 2020] when they are selected.

[b] Non-MRIO enacting metrics (Population and GDP) are treated as deterministic. For economic enacting metrics, uncertainty is limited to inter-MRIO variability and is represented by a continuous uniform distribution derived from differences across MRIO sources. For environmental enacting metrics, uncertainty is modeled using continuous uniform probability distributions based on regional coefficients of variation (CoVs) for consumption-based accounting of GHG emissions reported by [Rodrigues et al., 2018]. Since these CoVs have been estimated exclusively for greenhouse gas emissions, the same coefficients are applied uniformly across all impact categories.

References

[Aitchison, 1982] Aitchison, J. (1982). The statistical analysis of compositional data. *Journal of the Royal Statistical Society: Series B (Methodological)*, 44(2):139–160.

[Algunaibet et al., 2019] Algunaibet, I. M., Pozo, C., Galán-Martín, Á., Huijbregts, M. A. J., Dowell, N. M., and Guillén-Gosálbez, G. (2019). Powering sustainable development within planetary boundaries. *Energy & Environmental Science*, 12(6):1890–1900.

[Bai et al., 2024] Bai, X., Hasan, S., Andersen, L. S., Bjørn, A., Kilkiş, Ş., Ospina, D., Liu, J., Cornell, S. E., Sabag Muñoz, O., de Bremond, A., Crona, B., DeClerck, F., Gupta, J., Hoff, H., Nakicenovic, N., Obura, D., Whiteman, G., Broadgate, W., Lade, S. J., Rocha, J., Rockström, J., Stewart-Koster, B., van Vuuren, D., and Zimm, C. (2024). Translating Earth system boundaries for cities and businesses. *Nature Sustainability*, 7(2):108–119.

[Balanza et al., 2025] Balanza, T., Campion, N., Berg Rosendal, M., Bramstoft, R., and Bjørn, A. (2025). Exploring configurations of a European energy system within the planetary boundaries. *Environmental Research Letters*, 20(7):074050.

[Beaufils and Wenz, 2022] Beaufils, T. and Wenz, L. (2022). A scenario-based method for projecting multi-regional input–output tables. *Economic Systems Research*, 34(4):440–468.

[Björklund, 2002] Björklund, A. E. (2002). Survey of approaches to improve reliability in lca. *The International Journal of Life Cycle Assessment*, 7(2):64–72.

[Bjørn et al., 2020] Bjørn, A., Chandrakumar, C., Boulay, A.-M., Doka, G., Fang, K., Gondran, N., Hauschild, M. Z., Kerkhof, A., King, H., Margni, M., McLaren, S., Mueller, C., Owsianiak, M., Peters, G., Roos, S., Sala, S., Sandin, G., Sim, S., Vargas-Gonzalez, M., and Ryberg, M. (2020). Review of life-cycle based methods for absolute environmental sustainability assessment and their applications. *Environmental Research Letters*, 15(8):083001.

[Bjørn et al., 2026] Bjørn, A., Fantke, P., Jolliet, O., Laurent, A., Owsianiak, M., Ryberg, M., Hauschild, M., and Vea, E. B. (2026). Beyond net zero climate targets: A research agenda for absolute environmental sustainability assessment to support decisions at different scales. *Environmental Research Letters*.

[Bjørn and Hauschild, 2015] Bjørn, A. and Hauschild, M. Z. (2015). Introducing carrying capacity-based normalisation in lca: framework and development of references at midpoint level. *The International Journal of Life Cycle Assessment*, 20:1005–1018. `https://doi.org/10.1007/s11367-015-0899-2`.

[Bjørn et al., 2023] Bjørn, A., Lloyd, S., Schenker, U., Margni, M., Levasseur, A., Agez, M., and Matthews, H. D. (2023). Differentiation of greenhouse gases in corporate science-based targets improves alignment with Paris temperature goal. *Environmental Research Letters*, 18(8):084007.

[Bjørn et al., 2025] Bjørn, A., Paulillo, A., Sanye Mengual, E., De Laurentiis, V., Vea, E., Hauschild, M. Z., and Sala, S. (2025). *Guidance for Applying Absolute Environmental Sustainability Assessment on Activities at Different Scales (BETA Version)*. Publications Office of the European Union.

[Cap et al., 2025] Cap, S., Li, S., de Koning, A., Karjalainen, A., Lettenmeier, M., Coscieme, L., Tukker, A., and Scherer, L. (2025). Carbon footprint reduction potential of consumption changes in five European countries in 2015, 2030, and 2050. *Sustainable Production and Consumption*, 59:408–421.

[Clausen et al., 2025] Clausen, C. A., Hauschild, M. Z., and Bjørn, A. (2025). Absolute environmental sustainability assessments of long-lived systems: A review of challenges with the representation of time and future research directions. *Sustainable Production and Consumption*. `https://doi.org/10.1016/j.spc.2025.06.006`.

[Gebara and Laurent, 2023] Gebara, C. H. and Laurent, A. (2023). National sdg-7 performance assessment to support achieving sustainable energy for all within planetary limits. *Renewable and Sustainable Energy Reviews*, 173:112934. https://doi.org/10.1016/j.rser.2022.112934.

[Gopalakrishnan, 2022] Gopalakrishnan, S. (2022). The why and how of assigning responsibility for supply chain emissions. *Nature Climate Change*, 12(12):1075–1077.

[Gopalakrishnan et al., 2021] Gopalakrishnan, S., Granot, D., Granot, F., Sošić, G., and Cui, H. (2021). Incentives and Emission Responsibility Allocation in Supply Chains. *Management Science*, 67(7):4172–4190.

[Greene, 2003] Greene, W. H. (2003). *Econometric Analysis*. Prentice Hall, Upper Saddle River, NJ, 5 edition.

[Hauschild et al., 2018] Hauschild, M. Z., Rosenbaum, R. K., and Olsen, S. I., editors (2018). *Life Cycle Assessment: Theory and Practice*. Springer International Publishing, Cham.

[Hjalsted et al., 2021] Hjalsted, A. W., Laurent, A., Andersen, M. M., Olsen, K. H., Ryberg, M., and Hauschild, M. (2021). Sharing the safe operating space: Exploring ethical allocation principles to operationalize the planetary boundaries and assess absolute sustainability at individual and industrial sector levels. *Journal of Industrial Ecology*, 25(1):6–19.

[Huijbregts, 1998] Huijbregts, M. A. J. (1998). Application of uncertainty and variability in LCA. *The International Journal of Life Cycle Assessment*, 3(5):273–280.

[IIASA, 2024] IIASA (2024). SSP Scenario Explorer, v3.1.0.

[International Institute for Applied Systems Analysis, nd] International Institute for Applied Systems Analysis (n.d.). Shared Socioeconomic Pathways Scenario Explorer hosted by IIASA. https://data.ece.iiasa.ac.at/ssp/. Data portal. Accessed: 2026-06-04.

[InternationalMonetaryFund, 2025] InternationalMonetaryFund (2025). World econimic outlook.

[Kromand et al., 2025] Kromand, J. B., Tilsted, J. P., and Bjørn, A. (2025). Developing sufficiency-based sharing principles for absolute environmental sustainability assessment using decent living standards and planetary boundaries. *Sustainable Production and Consumption*, 54:516–529.

[Lejeune et al., 2026] Lejeune, M., Kara, S., Hauschild, M. Z., Shahrabifarahani, S., and Daiyan, R. (2026). Pathways to global hydrogen production within planetary boundaries. *Nature Communications*, 17(1):3521.

[Lenzen et al., 2013] Lenzen, M., Moran, D., Kanemoto, K., and Geschke, A. (2013). Building Eora: a global multi-region input–output database at high country and sector resolution. *Economic systems research*, 25(1):20–49. https://doi.org/10.1080/09535314.2013.769938 (accessed June 2026).

[Lenzen and Murray, 2010] Lenzen, M. and Murray, J. (2010). Conceptualising environmental responsibility. *Ecological Economics*, 70(2):261–270.

[Lenzen et al., 2007] Lenzen, M., Murray, J., Sack, F., and Wiedmann, T. (2007). Shared producer and consumer responsibility — Theory and practice. *Ecological Economics*, 61(1):27–42.

[Lucas et al., 2020] Lucas, P. L., Wilting, H. C., Hof, A. F., and van Vuuren, D. P. (2020). Allocating planetary boundaries to large economies: Distributional consequences of alternative perspectives on distributive fairness. *Global Environmental Change*, 60:102017.

[Mendoza Beltran et al., 2016] Mendoza Beltran, A., Heijungs, R., Guinée, J., and Tukker, A. (2016). A pseudo-statistical approach to treat choice uncertainty: The example of partitioning allocation methods. *The International Journal of Life Cycle Assessment*, 21(2):252–264.

[OECD, 2023] OECD (2023). Inter-Country Input-Output tables. https://www.oecd.org/en/data/datasets/inter-country-input-output-tables.html.

[OECD, 2025] OECD (2025). Inter-Country Input-Output tables. https://www.oecd.org/en/data/datasets/inter-country-input-output-tables.html.

[Oosterhoff et al., 2023] Oosterhoff, H. C., Golsteijn, L., Laurent, A., and Ryberg, M. W. (2023). A new consistent framework for assignment of safe operating space to B2C and B2B industries for use in absolute environmental sustainability assessments. *Journal of Cleaner Production*, 399:136574.

[Paulillo and Sanyé-Mengual, 2024] Paulillo, A. and Sanyé-Mengual, E. (2024). Approaches to incorporate Planetary Boundaries in Life Cycle Assessment: A critical review. *Resources, Environment and Sustainability*, 17:100169.

[Paulillo et al., 2026] Paulillo, A., Wierzgala, P., Biganzoli, F., and Sanyé-Mengual, E. (2026). Investigating methodological aspects of absolute environmental sustainability assessment based on planetary boundaries. *Environmental Impact Assessment Review*, 117:108154.

[Pawlowsky-Glahn et al., 2015] Pawlowsky-Glahn, V., Egozcue, J. J., and Tolosana-Delgado, R. (2015). *Modelling and Analysis of Compositional Data*. John Wiley & Sons, Chichester, UK.

[Puig-Samper et al., 2025] Puig-Samper, G., Owsianiak, M., Clavreul, J., Jeandaux, C., Prieur-Vernat, A., and Gondran, N. (2025). Quantifying uncertainties in absolute environmental sustainability assessment: A general framework applied to French electricity production. *Sustainable Production and Consumption*, 54:12–24.

[Richardson et al., 2023] Richardson, K., Steffen, W., Lucht, W., Bendtsen, J., Cornell, S. E., Donges, J. F., Drüke, M., Fetzer, I., Bala, G., Von Bloh, W., et al. (2023). Earth beyond six of nine planetary boundaries. *Science advances*, 9(37):eadh2458. `https://doi.org/10.1126/sciadv.adh2458`.

[Rodrigues et al., 2018] Rodrigues, J. F. D., Moran, D., Wood, R., and Behrens, P. (2018). Uncertainty of Consumption-Based Carbon Accounts. *Environmental Science & Technology*, 52(13):7577–7586.

[Rosenbaum et al., 2017] Rosenbaum, R. K., Georgiadis, S., and Fantke, P. (2017). Uncertainty management and sensitivity analysis. In *Life cycle assessment: theory and practice*, pages 271–321. Springer.

[Ryberg et al., 2020] Ryberg, M. W., Andersen, M. M., Owsianiak, M., and Hauschild, M. Z. (2020). Downscaling the planetary boundaries in absolute environmental sustainability assessments – A review. *Journal of Cleaner Production*, 276:123287.

[Ryberg et al., 2018] Ryberg, M. W., Owsianiak, M., Richardson, K., and Hauschild, M. Z. (2018). Development of a life-cycle impact assessment methodology linked to the Planetary Boundaries framework. *Ecological Indicators*, 88:250–262.

[Sacchi et al., 2022] Sacchi, R., Terlouw, T., Siala, K., Dirnaichner, A., Bauer, C., Cox, B., Mutel, C., Daioglou, V., and Luderer, G. (2022). PRospective EnvironMental Impact asSEment (*premise*): A streamlined approach to producing databases for prospective life cycle assessment using integrated assessment models. *Renewable and Sustainable Energy Reviews*, 160:112311.

[Sala et al., 2020] Sala, S., Crenna, E., Secchi, M., and Sanyé-Mengual, E. (2020). Environmental sustainability of european production and consumption assessed against planetary boundaries. *Journal of environmental management*, 269:110686. `https://doi.org/10.1016/j.jenvman.2020.110686`.

[Sanye Mengual and Sala, 2023] Sanye Mengual, E. and Sala, S. (2023). Consumption footprint and domestic footprint: Assessing the environmental impacts of eu consumption and production. Scientific analysis or review KJ-NA-31-390-EN-N (online),KJ-NA-31-390-EN-C (print), Luxembourg (Luxembourg).

[Seber and Lee, 2003] Seber, G. A. F. and Lee, A. J. (2003). *Linear Regression Analysis*. Wiley Series in Probability and Statistics. Wiley-Interscience, Hoboken, NJ, 2 edition.

[Shukla et al., 2022] Shukla, P. R., Skea, J., Reisinger, A. R., and IPCC, editors (2022). *Climate Change 2022: Mitigation of Climate Change*. IPCC, Geneva.

[Södersten et al., 2018] Södersten, C.-J. H., Wood, R., and Hertwich, E. G. (2018). Endogenizing Capital in MRIO Models: The Implications for Consumption-Based Accounting. *Environmental Science & Technology*, 52(22):13250–13259.

[Stadler, 2021] Stadler, K. (2021). Pymrio – A Python Based Multi-Regional Input-Output Analysis Toolbox. *Journal of Open Research Software*, 9(1).

[Stadler et al., 2018] Stadler, K., Wood, R., Bulavskaya, T., Södersten, C.-J., Simas, M., Schmidt, S., Usubiaga, A., Acosta-Fernández, J., Kuenen, J., Bruckner, M., Giljum, S., Lutter, S., Merciai, S., Schmidt, J. H., Theurl, M. C., Plutzar, C., Kastner, T., Eisenmenger, N., Erb, K.-H., de Koning, A., and Tukker, A. (2018). EXIOBASE 3: Developing a Time Series of Detailed Environmentally Extended Multi-Regional Input-Output Tables. *Journal of Industrial Ecology*, 22(3):502–515.

[Stadler et al., 2026] Stadler, K., Wood, R., Bulavskaya, T., Södersten, C.-J., Simas, M., Schmidt, S., Usubiaga, A., Acosta-Fernández, J., Kuenen, J., Bruckner, M., Giljum, S., Lutter, S., Merciai, S., Schmidt, J. H., Theurl, M. C., Plutzar, C., Kastner, T., Eisenmenger, N., Erb, K.-H., and Tukker, A. (2026). EXIOBASE 3 (3.10.2). Data set.

[Steffen et al., 2015] Steffen, W., Richardson, K., Rockström, J., Cornell, S. E., Fetzer, I., Bennett, E. M., Biggs, R., Carpenter, S. R., De Vries, W., De Wit, C. A., et al. (2015). Planetary boundaries: Guiding human development on a changing planet. *science*, 347(6223):1259855. `https://doi.org/10.1126/science.1259855`.

[van den Berg et al., 2020] van den Berg, N. J., van Soest, H. L., Hof, A. F., den Elzen, M. G. J., van Vuuren, D. P., Chen, W., Drouet, L., Emmerling, J., Fujimori, S., Höhne, N., Kõberle, A. C., McCollum, D., Schaeffer, R., Shekhar, S., Vishwanathan, S. S., Vrontisi, Z., and Blok, K. (2020). Implications of various effort-sharing approaches for national carbon budgets and emission pathways. *Climatic Change*, 162(4):1805–1822.

[Vea et al., 2020] Vea, E. B., Ryberg, M., Richardson, K., and Hauschild, M. Z. (2020). Framework to define environmental sustainability boundaries and a review of current approaches. *Environmental Research Letters*, 15(10):103003. `https://iopscience.iop.org/article/10.1088/1748-9326/abac77/meta`.

[Verhaeghe et al., 2024] Verhaeghe, R., Mouton, L., Trigaux, D., and Allacker, K. (2024). Carrying capacity-based benchmarks for Belgian residential buildings. *Journal of Environmental Management*, 370:122914.

[Wiebe et al., 2018] Wiebe, K. S., Bjelle, E. L., Többen, J., and Wood, R. (2018). Implementing exogenous scenarios in a global MRIO model for the estimation of future environmental footprints. *Journal of Economic Structures*, 7(1):20.

[Wooldridge, 2009] Wooldridge, J. M. (2009). *Introductory Econometrics: A Modern Approach*. South-Western Cengage Learning, Mason, OH, 4 edition.

[WorldBank, 2025] WorldBank (2025). World development indicators.

[Yamano et al., 2023] Yamano, N., Alsamawi, A., Webb, C., Cimper, A., Zürcher, C., and Pechansky, R. C. (2023). Development of the OECD Inter Country Input-Output Database 2023. *OECD Science, Technology and Industry Working Papers*, 2023(08).

[Yang and Paulillo, 2025] Yang, Q. and Paulillo, A. (2025). Advancing Planetary Boundaries Allocation: Systematic Comparison of Sharing Principles for National-Level Absolute Environmental Sustainability Assessments. *Procedia CIRP*, 135:875–880.